MODEL-PROTOTYPE CONFORMITY
FOR PREDICTING SCOUR IN HYDRAULIC STRUCTURES

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ABSTRACT
Prediction of scour in hydraulic structures is necessary for their safety and economy. In India, scour in almost all hydraulic structures built on rivers and canals, is computed by Lacey’s regime formula as prescribed in Indian codes. Scour in hydraulic structures is a very complex process and it is governed by number of other parameters, besides discharge and mean size of bed materials used in Lacey’s scour model. In the developed countries, scour is determined by mathematical models developed on the basis of model study - both numerical and physical - after validation of the model with actual scour data observed in prototype structures. In this paper, author has discussed a few such models for scour estimation in bridges and barrages. Necessity of field data collection for model – prototype conformity of scour models has been emphasized.

INTRODUCTION
Almost all hydraulic structures constructed in rivers and canals are subject to scour. Protection against scour is essential for the safety of these structures, economy and reduction of their annual maintenance cost. Uncontrolled scour without requisite protective measures will result in their progressive or sudden failure, depending on the magnitude of flood the structures experience after their construction. Due to uncertainty of flood events, there is, however, always a risk involved and as such adequate safeguards have to be made against scour failure right at the construction stage.

Scouring occurs in alluvial channels due to natural morphological changes - commonly termed as general scour. Scour also occurs due to change in flow field triggered by the construction of hydraulic structures e.g. dams, barrages, bridges, culverts, intakes, groynes etc. on the river and the various regulating and control structures e.g. drop structures, cross-drainage structures etc in canals - termed as localized scour. Natural or general scour in rivers can cause dramatic changes in river plan, cross-section and course of a river, leading to outflanking of the structures, making them useless and some times avulsion of river requiring closure of breaches and training works at great cost in order to bring back the river to its original course. Localised scour occurs due to either constriction of waterway (called contraction scour) or obstruction of and interference with the structures - termed as local scour. Superimposition of general scour with localized scour gives total scour depth in hydraulic structures. All protection works are to be designed to take care of the total scour depth likely to occur in hydraulic structures.

Scouring is a complex process as to preclude efforts at full understanding and hence accurate preventive or protective measures. Any underestimation of scour may result in failure of structures, costly protective measures and annual maintenance. On the other hand, overestimation of scour will unnecessarily increase cost of the structure.

Despite much study, the principles of analysis of scouring are not well established. Being closely linked with turbulence and sediment transport, both of which are complex phenomena, exact numerical modeling of scour is extremely difficult. It is for this reason, physical modeling of hydraulic structures is necessary in understanding the scour mechanism and prediction of scour in prototype from observed scour in model.

However, scaled model testing to predict exact scour in prototype is almost impossible, since complete similarity between model and prototype can not be achieved in any scaled model.
test. It is usual to prepare a geometrically similar model since distortion of scale (Lr) may not reproduce the exact flow conditions (governing scour) in the model. The most common method of scour estimation, therefore, is to take help of both numerical and physical modeling simultaneously. Some of the mathematical models used in scour estimation in a few hydraulic structures are briefly discussed underneath.

**LACEY’S MODEL FOR SCOUR ESTIMATION**


\[
P = 4.8 \frac{Q^{1/2}}{f} \quad (1)
\]

\[
R = 0.475 \left( \frac{Q}{f} \right)^{1/3} \quad (2)
\]

\[
R= \frac{3}{8} \left( \frac{q^2}{f} \right)^{1/3} \quad (3)
\]

\[
f = 1.76 \sqrt{d_{50}} \quad (4)
\]

Eq 1 gives Lacey’s stable waterway (P in m) that a river/canal will develop (by scouring its bed and bank) as the flood of magnitude Q passes through it. Eq.2 gives the Lacey’s regime depth (R in m) which it will attain by scouring the bed. Eq.3 (Blench,1969) also gives the regime depth (R in m) when Lacey’s waterway is restricted due to fluming or contraction of normal/ regime waterway i.e. when the actual waterway (L) provided for the structure is less than Lacey’s waterway (P). Eqs. (2)&(3) give same R-value when L=P.

Local scour depth in guide bunds, spurs, piers, abutments etc. are found by multiplying R with factors varying from 1.25 to 2.5 (CBIP,1989), depending on location of scour. All Indian codes provide the values of these multiplying factors based on the experience of senior engineers (in Govt. and Pvt. Co.’s) associated with the preparation of codes/guidelines.

Limitations of Lacey’s theory in prediction of scour in bridges have been discussed by the author in an earlier paper (Mazumder,2007). Scour is a complex phenomenon and is governed by many other parameters, besides Q or q and d_{50}, as per Lacey's/Blench's equations. Lacey’s/Blench’s method of computing scour has most serious limitation because it does not consider non-uniformity of bed sediments and it does not consider live bed condition in a river after the threshold condition. As per these equations scour goes on increasing with increasing Q/q, which is incorrect after threshold/critical state. Scour computation by using mathematical models in some of the hydraulic structures are discussed briefly underneath.

**SCOUR IN BRIDGES**

Estimation of total scour depth comprising of general scour, contraction scour and local scour in bridges is extremely important for safe design of foundation of piers and abutments

**General Scour Model**

Lane (1957), Lacey (1930), Blench (1969), Neill (1973), Garde and Rangaraju (2000), Chitale (1981), Yalin (1993) have done commendable work to determine the general river behavior to find dimensions of stable channel section for propagation of floods. In India, the most popular method of predicting the stable channel dimension is by using Lacey’s regime equations. Most of these equations are, however, applicable for finding scoured bed profile of a river when the bed and bank material consist of fine alluvial material. By far the most difficult part in modeling is to simulate the general scour and prediction of river behavior (Mazumder,2004) in the vicinity of hydraulic structures. It needs continuous exchange of data between the field observations and laboratory results. Based on field and laboratory data, mathematical models and software e.g. HEC-models, have been developed for prediction of river behavior near hydraulic structures.

**Contraction/Constriction Scour Model**

Richardson and Davis (1995) have performed exhaustive study to determine contraction
scour in bridges. Based on their model study and validation with field data, the following models are used for computing scour depth in a constriction (HEC-18)

\[
Y_2 = 1.48 \frac{Q_2}{(d_m^{1/3}W_2)^{6/7}} \text{ for clear water scour } (\tau_o < \tau_c \text{ or } V < V_c), \quad \cdots \cdots \cdots \quad (5)
\]

\[
Y_2/Y_1 = \left(\frac{Q_2/Q_{1m}}{W_1/W_2}\right)^{6/7} K_1 \text{ for live bed conditions } (\tau_o > \tau_c \text{ or } V > V_c) \quad \cdots \cdots \cdots \quad (6)
\]

where \(Y_2\) is the average depth (including scour depth) under the bridge in meter, \(Y_1\) is the average depth of flow in the approach channel, \(Q_2\) is the total discharge through bridge in cumec, \(Q_{1m}\) is the discharge in that part of approach channel which transport bed load, \(d_m\) is effective mean diameter of the bed material in mm (\(d_m = 1.25 d_{50}\)), \(W_1\) and \(W_2\) are the mean widths of the stream in the approach channel and the contracted section under the bridge respectively. It is assumed that the scour continues to occur in the contracted reach until threshold condition is attained. Constriction scour depth (\(d_{sc}\)) measured below original river bed given by \(d_{sc} = (Y_2 - Y_1)\) in m, the constant (1.48) has a dimension (L^{-3/7}). \(K_1\) is a coefficient varying from 0.59 (for sediments transported mostly as bed load) to 0.69 (for sediment transport mostly in suspended form).

**Local Scour Models**

Local scour in bridge piers occur due to obstruction by piers and abutments and the consequent changes in the flow field around the piers and abutments of bridge. From non-dimensional analysis of the different parameters governing scour around a pier, it can be proved that

\[
d_s/b = f \left(\frac{V}{V_c}, \frac{y}{b}, \frac{b}{d_{50}}, \sigma_g, Sh, Al, G, \frac{Vt}{b}, \frac{V}{gb}\right) \quad \cdots \cdots \cdots \quad (7)
\]

First three terms represent flow intensity, flow shallowness and coarseness of sediments respectively, \(b\) is the thickness (size) of pier, \(\sigma_g\) is the geometric non-uniformity coefficient of sediments expressed as \((d_{84}/d_{16})^{0.5}\). \(Sh\) & \(Al\) are governed by the shape and alignment of piers, \(G\) represents the non-uniformity of approach flow and shape of cross-section of the approach channel, \(Vt/b\) is a non-dimensional time parameter representing the actual time of scour with respect to the time \((t_e)\) required to attain equilibrium scour depth \((d_{sc})\), and the last parameter gives Froude’s number of flow based on pier thickness, \(b\).

There are large numbers of research study on local scour around bridge piers all over the world and a large numbers of mathematical models have been evolved for estimating local scour around piers and abutments, principally on the basis of laboratory model study. In all mathematical models, total scour depth is found by adding up local scour with the general scour and the constriction (or contraction) scour as discussed above. Some of the most popular mathematical models which have been used to estimate local scour depth in bridge piers are briefly discussed in the following paragraphs

**Melville and Coleman Model**

Melville and Coleman (2000) method of estimation of local scour around piers (\(d_s\)) below river bed is given by equation (8):

\[
d_s = K_{yb} \cdot K_1 \cdot K_d \cdot K_a \cdot K_g \cdot K_t \quad \cdots \cdots \cdots \quad (8)
\]

All other parameters except \(K_{yb}\) are non-dimensional and \(K_{yb}\) is having the same dimension as that of \(d_s\) i.e. scour depth in meter. \(K_{yb}\) is depth-size or shallowness factor and is given by the relation \(K_{yb} = 2.4 b\) when \(b/y < 0.7, K_{yb} = 4.5y\) when \(b/y > 5\) and \(K_{yb} = 2 \sqrt{y/b}\) when \(0.7 < y/b < 5\). \(K_1\) is flow intensity factor including sediment gradation, \(K_d\) is sediment size factor, \(K_a\) is pier shape factor, \(K_g\) is pier alignment factor, \(K_g\) is channel geometry factor, \(K_t\) is the time factor. The different \(K\)-values, the various mathematical equations and the design curves are given in the book “Bridge Scour” by Melville and Coleman (2000). The model, used in Australia and New Zealand, has been validated with prototype scour data collected at several bridge sites in New Zealand and Australia.
HEC-18 Model (Richardson and Davis)

HEC-18 (Richardson and Davis, 1995) recommend use of the following equation for both clear water and live bed scour depth, $d_s$, (measured below bed) in terms of approach flow depth, $y_1$ as

$$d_s/y_1 = 2K_1 . K_2 . K_3 . K_4 (b/y_1)^{0.65} . Fr_1^{0.43} \quad \text{(9)}$$

where $K_1$ is correction factor for pier nose shape i.e. $K_s$ in Mellville equation, $K_2$ is correction factor for flow obliquity i.e. $K_{al}$ in Melville equation, $K_3$ is correction factor for bed condition i.e. plain bed, ripple and dune bed etc., $K_4$ is the correction factor due to armoring of bed in non-uniform sediments, $Fr_1$ is the approach flow Froude’s number directly upstream of pier given by the relation

$$Fr_1 = V_1/\sqrt{gy_1} \quad \text{(10)}$$

where $V_1$ is the mean velocity of flow and $y_1$ is the average flow depth directly upstream of piers. Values of $K_1$, $K_2$, $K_3$, $K_4$ are given in HEC-18 as well as in the book “Hydraulic Design Hand book” by Mays, (1999) in Chapter 15. This mathematical model, used in USA, has been proved with prototype bridge scour collected from different regions in USA.

IAHR Model (Breussers & Raudkivi)

Breussers and Raudkivi (1991) have differentiated between live bed scour and clear water scour up to threshold condition. Equilibrium condition reaches when the combined effect of the temporal mean shear stress, the weight component and the turbulent forces are in equilibrium everywhere within the scour hole. For live bed scour, an excess shear stress $(\tau_o - \tau_c)$ must exist for transport of the sediments through the scour hole. However, the particles on the surface of the equilibrium scour hole may occasionally move but are not carried away.

For clear water, local scour ($d_w$) when $u < u_c$, or $V < V_c$

$$d_w/b = 2.3 K_\sigma K_{(b/d_{50})} K_d K_s K_\alpha \quad \text{(11)}$$

and for live bed scour when $u > u_c$, or $V > V_c$, the equation is

$$d_w/b = X . K_{(b/d_{50})} K_d K_s K_\alpha \quad \text{(12)}$$

Here, $d_w$ is the equilibrium scour depth measured below river bed, $K_\sigma$ is a coefficient for gradation of sediment, $K_{(b/d_{50})}$ is a coefficient owing to size of sediments with respect to pier size ‘b’, $K_d$ is a factor due to depth of flow or flow shallowness factor, $K_s$ is shape factor, $K_\alpha$ is the pier alignment factor. Maximum value of $X$ is 2.3 when $V > 4V_c$. When $V_c < V < 4V_c$, value of $X$ varies from 2 to 2.30 for uniform sediments ($\sigma_g \leq 1.3$) and “X” varies from 0.5 to 2.0 for non-uniform sediments ($\sigma_g > 1.3$). Known as IAHR model, it is extensively used in Europe for determining local scour in piers after verification with scour data collected from bridge sites.

Kothyari – Garde - Rangaraju Model

Based on the analysis of extensive laboratory data collected for uniform, non-uniform and stratified sediments, steady and unsteady flows, the following mathematical models have been proposed by Kothyari, Garde and Ranga Raju (1992) for estimation of local scour in bridge piers under clear water and live bed conditions when the flow is parallel to pier axis without any obliquity.

For clear water scour depth ($d_{cw}$) measured below river bed:

$$d_{cw}/d_{50} = 0.66(b/d_{50})^{0.75} \left\{ (D/d_{50})^{0.16} \right\} \left\{ (V^2-V_{c}^2)\rho/\Delta \gamma_s d_{50} \right\} \cdot a^{-0.30} \quad \text{(13)}$$

For live or mobile bed scour:

$$d_{cw}/d_{50} = 0.88 (b/d_{50})^{0.67} (D/d_{50})^{0.4} \cdot a^{-0.3} \quad \text{(14)}$$

where, $D$ is the average flow depth, $d_{50}$ is the mean sediment size, $V$ is the mean flow velocity, $\Delta \gamma_s = (\gamma_s - \gamma_f)$, $\gamma_s$ and $\gamma_f$ are the unit weights of sediments and water respectively, $\rho$ is the density of water, $a = (B-b)/B$, $B$ is the centre to centre spacing of piers, $b$ is the pier
thickness, $V$ is the actual mean velocity of flow under the bridge, $V_c$ is the mean critical velocity of flow at threshold condition of bed motion.

Unfortunately, the model is not incorporated in Indian codes so far as there is hardly any scour data observed at bridge sites in India for the purpose of validation under Indian river conditions.

Author (Mazumder, 2006) computed total depth of scour in piers in five major bridges in India by using the above models (Eqs. 8, 9, 11, 12, 13 & 14) and compared the results obtained with /IRC/IS/RDSO method of scour estimation based on Lacey’s theory (Eqs. 3 & 4). It is observed that Lacey’s method overestimates scour in all the cases and the error is found to vary from 5% to 255%. Scour depths obtained by the different mathematical models, other than Lacey’s model, is found to be almost the same.

**SOME MODELS OF SCOUR DOWNSTREAM OF WEIRS & BARRAGES**

Novak (1961) found that use of stilling basin which is sufficiently long to contain the jump will reduce the scour to some 45% to 65% of that without a stilling basin. Scour estimation is necessary for determining depth of cut-off and design of flexible protection works downstream of stilling basins.

Is Code (IS 6966, part-I, 1989) recommends use of Lacey’s equation (2) or (3) to estimate $R$, depending on looseness factor. Maximum scour depth below design high flood level is recommended as 1.75 $R$ for the design of downstream cut-off.

Many projects, especially the larger and complex ones, require model study - both numerical and physical. Numerical models can be made only when the physical process and the parameters governing scour are well understood and theoretical structure can be made based on systematic laboratory investigation and due verification with field data. Techniques for assuring acceptable similarity between model and prototype are based on following three principal criteria, namely,

(i) Geometric similarity (without distortion) between model and prototype

(ii) Dynamic similarity on the basis of Froude law, with such model scale so as to ensure turbulent flow in model.

(iii) Similarity of sediment transport characterized by making $S_u = S_{uc}$, where $S_u$ is the velocity scale as per Froude’s law and $S_{uc}$ is the velocity scale corresponding to critical velocity $U_c$ at which the bed starts moving in model and prototype i.e.

$$S_{uc} = U_{cp} / U_{cm} = L_{r}^{1/2}.$$ 

de Graauw, & Pyilarczyk(1981) conducted prototype scour tests under controlled conditions and repeated in a small scale model (1:30) with polysterene as bed material using similar ratios of $U/U_c$ in model and prototype, where $U$ is the mean flow velocity and $U_c$ is the critical velocity at threshold condition.

Results of model tests can be extrapolated to prototype type scale provided the time scale of the process is specified. The time development of the scour hole can be correlated as

$$d_{max} / Y_0 = f(t/t_1) = (t/t_1)^p$$  \hspace{1cm} (16)

for two dimensional scour, values of $p$ range from 0.2 (for scour below ogee type spillway) to 0.4 (for scour downstream of horizontal bed), $t$ is the time required to attain a scour depth $d_{max}$ (below original bed) at a time $t$ and $t_1$ is the time required to attain a scour depth $d_{max} = Y_0$ and $Y_0$ is the depth of flow in the channel before scour. The characteristics time $t_1$ found by Breusers (1967) after performing 250 experiments is

$$t_1 = 330 \Delta 1.7 Y_0^2 (a U - U_c)$$  \hspace{1cm} (17)

where, $\Delta = (\gamma_s - \gamma_w) / \gamma_w$, $a$ is a factor dependent on flow geometry found to vary between 2 to 3 depending on basin length and type of protection, $U$ is mean velocity at the end of basin and $U_c$ is critical mean velocity for bed materials at threshold condition of bed motion.
Mazumder and Praveen Kumar (1995) found that length of stilling basin (actually provided) compared to length of hydraulic jump and turbulence level at the end of basin have predominant effect on scour.

Bajestan et al. (1995) investigated depth of scour hole downstream of a SAF stilling basin. Analysing the forces causing dislodgement of bed particles, they developed a stability no SN given by equation (18).

\[ SN = \frac{V_1}{\sqrt{g (G_s - 1) D_{50}}} \]  

(18)

By regression analysis of data obtained from laboratory and field study, they established a non-dimensional correlation between maximum scour depth (\(d_s\)) measured below original bed, SN and prejump depth (\(y_1\)) given by equation (19).

\[ \frac{d_s}{D_{50}} = 0.0158 (SN)^{2.321} (\frac{y_1}{D_{50}})^{0.344} \]  

(19)

where \(V_1\) and \(Y_1\) are the prejump velocity and prejump depth respectively, \(D_{50}\) is mean size of bed sediments, \(G_s\) id the specific gravity of sediments

Mao Changxi (1995) proposed a general scour equation for finding maximum scour depth in different types of hydraulic structures in China. Equation (20) given below was derived by him after lengthy derivation of the basic equations governing scour and model tests.

\[ T = \Psi \frac{\sqrt{q (2\alpha - y/h)}}{\sqrt{(G_s - 1) g d_{90} \cos \beta (h/d_{90})^{1/6}}} \]  

(20)

where, \(T\) is the maximum scour depth below tail water level in meter, \(q\) is discharge intensity in cumec per meter width of channel, \(\alpha\) is the Corriolis coefficient , \(h\) is the tail water depth in m above original bed, \(y\) is the depth in m above the original bed at which velocity is maximum just after the basin also in m, \(G_s\) is the specific gravity of bed materials, \(d_{90}\) is the size of bed material in m than which 90% is finer by weight, \(\beta\) is the angle of impingement of the inflowing jet with horizontal. Mao Changxi (1995) has given the values of \(\Psi\), \(\sqrt{(2\alpha - y/h)}\) and \(\beta\) for different types of hydraulic structures in his paper.

**EXAMPLE OF SCOUR COMPUTATION BY DIFFERENT MODELS**

Scour depths computed by using equations (3), (19) & (20) with data obtained from a barrage are given below:

- \(q = 23.5\) cumec/m
- \(D_{50} = 25\) mm = .025 m, \(D_{90} = 90\) mm = 0.09 m
- Pre-jump depth, \(y_1 = 2.5\) m
- Post jump depth, \(y_2 = 5.56\) m

**(a ) By Lacey /IS /IRC method (Eq. 3 )**

Lacey’s \(f = 1.76 \sqrt{d_{50}} = 8.8\)

With 30% flow concentration (as per code), \(q = 1.3*23.5 = 30.55\) m²/sec as per code

\(R = 1.35 \left( \frac{q^2}{f} \right)^{1/3} = 6.37\) meter

Maximum depth of scour below tail water surface, \(d_s + y_2 = 1.75 R = 11.15\) m

Maximum scour depth, below bed = 11.15 – 5.56 = 5.69 m

**(b ) By Mao Changxi Equation (Eq. 19)**

\(h = y_2 = 5.56\) m, \(\Psi = 1.02\), \((2\alpha - y/h)^{1/2} = 1.3\), \(\beta = 0^\circ\) (values of \(\Psi\), \(\sqrt{(2\alpha - y/h)}\) and \(\beta\) are taken from table given by Changxi for rip-rap bed protection)

\[ T = \Psi \frac{\sqrt{q (2\alpha - y/h)}}{\sqrt{(G_s - 1) g d_{90} \cos \beta (h/d_{90})^{1/6}}} \]

\[ = (1.02*23.5*1.30)/[(1.65*9.8*0.09)^{1/2}*(5.56/0.09)^{1/6}] = 13.02 m \]

\(d_s = T - y_2 = 13.02 - 5.56 = 7.46\) m

**(c ) By Stability No.(SN) : Bajestan et al**

\[ SN = \frac{V_1}{\sqrt{g (G_s - 1) D_{50}}} \]

\(V_1 = \frac{q}{y_1} = 23.5 / 2.5 = 9.4\) m/sec

\(SN = 9.4 / (9.8*1.65*0.025) = 14.7\)

\(d_s / D_{50} = 0.0158 (SN)^{2.321} (y_1 / D_{50})^{0.344}\)

\[ = 0.0158 * (14.7)^{2.321} * (2.5 / 0.025)^{0.344} = 39.44 \]

\(d_s = 39.44 * 0.025 = 0.986\) m.
Considering the gravel and bolder bed at the barrage site, 2 m depth concrete cut-off was provided considering that 12m long flexible protection was provided after the stilling basin.

**NEED FOR COLLECTION OF SCOUR DATA FROM SITE**

It is extremely difficult to conclude which of the methods is correct unless the models are duly validated by actual depth of scour measured at the site during the passage of design flood. Such field data are rarely available from sites in India, although large numbers of hydraulic structures are constructed and maintained almost every year after flood at a huge cost. Similar is the case in regard to construction and maintenance of river training, bridges, culverts and other hydraulic structures.

In USA and other developed countries, scour and other data have been measured at numerous bridge sites by using sophisticated equipments like ADCP, ADV, Radar, GPS mounted on unmanned remote controlled boats with a view to validate the mathematical models (e.g. HEC-18 model). Unfortunately, no such effort has been made in India to verify Lacey’s model being extensively used for scour computation in almost every hydraulic structure irrespective of type of river bed material, geometric and flow conditions in different rivers. There is a great need of field data collection for proving the model and establish model-prototype conformity. Indian codes in regard to scour estimation should be updated in the light of model study and the mathematical models developed on the basis of laboratory results and duly validated by scour observed in numerous hydraulic structures in our country.

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