INSTABILITY OF FLOW DOWNS REAL. HYDRAULIC STRUCTURES

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ABSTRACT

Residual Kinetic energy of flow is defined as the difference in energy to be dissipated and that actually dissipated. Higher is the residual kinetic energy, greater is the non-uniformity and instability of flow downstream. Analytical & experimental study of flow through an expanding passage indicated that substantial amount of residual kinetic energy left downstream when the floor of the expansion was level, resulting in highly non-uniform and unstable flow. It was possible to eliminate the residual kinetic energy by providing reverse slope to floor of the expansion. When the floor of the expansion was sloped with optimum angle of inclination determined theoretically, the flow downstream was found to be uniform and stable. It is concluded therefore, that the instability of flow arises due to residual kinetic energy of flow.
KEYWORDS: Residual K.E., Expansion, Instability.

1. INTRODUCTION

A number of hydraulic structures such as barrages, bridges, embankments, groynes etc. are constructed in alluvial flood plain of river to meet various objectives like storage and diversion of flow, communication, flood control, river training etc. In most of such constructions, original flood plain of the river is constricted. Fig. 1(B) shows the plan view of Kosi barrage in India. There is a lateral constriction of flood plain of 6,900 m width to 1150 m, width of Kosi barrage. Apart from lateral constriction, there is constriction of flow in the vertical plain also (Fig.1A). Owing to these constrictions, there is an afflux (3.6 m in Kosi barrage), resulting in rise of total energy line (TEL) upstream with respect to the normal TEL downstream prior to the construction.

For structures like dams and barrages, there is provision for dissipation of energy. No such provision exists in case of bridges, embankments, groynes etc. No energy dissipating structure can be designed for 100% dissipation of energy (ΔE in Fig.1A) under all conditions of flow. As such some residual (K.E.) of flow (ΔE - ΔE), as shown in Fig. 1A, leaves all such hydraulic
structures downstream. The excess (or residual) K.E. of flow can be contained in a given flow with a given depth only through non-uniform distribution of velocity.

![Diagram showing section of Kosi Barrage](image)

Fig. 2A showing section of Kosi Barrage

![Diagram showing plan of Kosi River Barrage](image)

Fig. 2B showing plan of Kosi River Barrage

K.E. correction factor (also known as Coriolis’ coeff.) defined by eq.1 below is a measure of distortion and non-uniformity of flow

\[ \alpha_2^2 = \frac{1}{A_2 v_2^3} \int \frac{u^3}{A_2} \, dA \]  \hfill (1)

where \( \alpha_2 \) is the K.E. correction factor downstream, \( A_2 \) & \( v_2 \) are the downstream cross-sectional area and mean velocity of flow respectively, \( u \) is the local velocity of flow through an elementary area \( dA \) on \( A_2 \)-plane.

Instability of flow may occur when \( \alpha_2 \) is very high, indicating that substantial amount of residual K.E. is transported downstream. For normal and uniform flow \( \alpha_2 = 1.0 \). Since the normal K.E. of flow \( (v_2^3/2g) \) in sub-critical flow is extremely small in most of the rivers flowing in alluvial flood plains, even a small amount of residual K.E. may cause large amount of distortion of flow resulting in high value of \( \alpha_2^2 \). Relation between efficiency of an energy dissipator \( \eta (= AE/\Delta E) \) and \( \alpha_2 \) is plotted in Fig. 2. It is seen that even 2% of residual K.E. (\( \eta = 98% \)) results in \( \alpha_2 \)-value ranging between 4 to 14 depending on flow condition.

![Graph showing relation between energy efficiency and K.E. correction factor](image)

Fig. 2: Relation of efficiency (\( \eta \)) with K.E. correction factor (\( \alpha_2 \))

Non-uniformity of flow associated with instability is the root cause of erosion, meandering and even change of river course downstream of hydraulic structures, constructed in alluvial flood plains. Such phenomena are reported to have occurred downstream of barrages like Kosi, Gandak & Farakka etc. in India.
INSTABILITY OF FLOW DOWNSTREAM OF HYDRAULIC STRUCTURES.

One of the Principal objectives of the paper is to investigate the Problem of instability of flow downstream of hydraulic structures.

2. REVIEW OF EARLIER WORKS

Sherednov (1967) attributed instability of flow due to sudden expansion resulting in formation of eddies. From the momentum, energy & continuity Principles, he developed the following criteria for instability of flow. The flow will be unstable if

\[ -\frac{dQ_c}{ds} < \frac{V_c}{\alpha_b} \left[ \lambda_g \chi_c + \frac{\lambda_k h_c}{\beta} \right] \]  

(2)

where, \( \lambda_g \) is the coefficient of eddy viscosity, \( \lambda_k \) is the coeff. of molecular viscosity, \( \alpha_b \) is the Boussinesq's coefficient, \( \chi_c \) is the perimeter of the channel in the region of central live stream, \( V_c \) is the mean velocity of flow and \( Q_c \) is the rate of flow of the live Central stream at any distance \( S \) from the entry to the expansion. Indices 1, 2, 3 refer to eddy flow on the left & right sides of the Central stream respectively. \( -\frac{dQ_c}{ds} \) is the rate of flow exchange from the live stream to the eddies in the rear half of the eddies. For any given flow (\( V_c \)), Eq.2 states that the instability of flow is governed by the following parameters:

(a) Rate of flow exchange \( -\frac{dQ_c}{ds} \); higher is the exchange rate, greater is the instability.

(b) Non-Uniformity of flow (\( \alpha_b \)); higher the non-uniformity, greater will be \( \alpha_b \) and more will be the instability.

(c) Resistance offered by turbulent shear \( \lambda_g \) and viscous shear \( \lambda_k \); higher the values of \( \lambda_g \) & \( \lambda_k \), more stable will be the flow.

Kline (1959) and Smith & Layne (1979) studied development of stall (separating flow in eddy) in 2-D diffusers. The important parameters which govern the different flow regimes & stability are:

(a) Length of diffuser wall (L) and the width (W) at entry to the diffuser; higher the value of L/W, more unstable is the flow.

(b) Total angle of the diffuser (2\( \Theta \)); greater the 2\( \Theta \)-value more unstable is the flow.

(c) Turbulence level of the incoming approach flow; higher the initial turbulence level, more stable will be the flow.

3. INVESTIGATION BY THE AUTHORS (1954)

3.1 Flow Characteristics In Expansion With Level Bed (\( \beta = 0 \)).

3.1.1 Momentum & energy principles in expansion with level bed.

Assuming linear variation of water surface in an expansive passage (Fig.3) the axial component of side wall reactions \( 2 \bar{R}_{sx} \) may be expressed as

\[ 2 \bar{R}_{sx} = \frac{V}{\beta} \left[ y_1^2 + y_2^2 + y_1 y_2 \right] \]  

(3)

where, \( \bar{y}_d \) is the unit wt. of water; various other symbols used are explained in Fig.3. Entering the above expression in momentum equation, theoretical relation between the depth ratio (\( y' = y_e/y_0 \)) is given by

\[ F_d^2 = y' \left( 2 - 2 y'^2 + y'^2 + y - y' - y' y'^2 \right) / (1 - y') \]  

(4)
where, \( F_b \) is the Froude's number of flow at entry to the expansion, \( r \) is the constriction ratio \( (r = B/b) \).

\[ \Delta E/\gamma_1 = (1 - y') - \frac{(1 + y'y)(2 + 2y'y + y'r + r - y' - y'^2)}{12y'r} \] (5)

Solid lines in Fig. 4 give the variation of \( \Delta E/\gamma_1 \) with \( F_b \) as obtained from eq. 4 and eq. 5. It may be seen that \( \Delta E/\gamma_1 \) increase with \( F_b \) & \( r \) (i.e., \( 2\theta \) for a given value of \( b \)). Residual Kinetic energy of flow, \( (\Delta E - \Delta E)/\gamma_1 \) arises when the actual loss of energy \( \Delta E/\gamma_1 \) due to production of turbulence lags behind \( \Delta E/\gamma_1 \) required in a level bed.

3.1.2 Experimental results:
Authors studied the flow characteristics in the expansion with level bed \( (\theta = 0^\circ) \) as shown in Fig. 3. It was observed that the flow remained unstable upto a certain angle \( (\theta = 20^\circ) \) of the expansion. With further increase in \( \theta \), the flow became unstable resulting in non uniformity of velocity distribution, separa-
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Values were computed from the measured distribution of velocity in each case and are given in line (iii) of Table 1. It may be seen that with increase in $2\Theta$, values of $\alpha_2$ increased rapidly—indicating thereby transport of substantial amount of residual K.E. downstream. Non-dimensional values of residual K.E. of flow, $(\Delta E - \Delta E')/Y_1$, obtained experimentally are given in line (v) of Table 1.

Fig. 5 separation and formation of eddy on the left side, $2\Theta = 20^\circ, \Psi = 0^\circ$

Fig. 6 Separation with eddies on either side and central unstable jet flow, $2\Theta = 60^\circ, \Psi = 0^\circ$

3.2 Flow Characteristics in Expansion With Inclined Dred

3.2.1 Optimum angle of Inclination ($\beta_{opt}$)

Fundamental difference between flow characteristics in a passage having parallel side walls (with stable flow) and that in an expansive passage having diverging side walls (with unstable flow) lies in the fact that the axial component ($R_{Sx}$) of side wall reaction in the latter, does not exist in the former. It was contemplated, therefore, that the flow in an expansion may be stabilised by providing reverse slope to the floor of the expansion such that the axial force ($R_{bx}$ in Fig. 3) acting backward exactly balanced the axial force ($2R_{Sx}$) acting in the forward direction. It may be shown that the value of $R_{bx}$ is given by

$$ R_{bx} = \frac{YwL \tan \beta}{b} (bY_2 + BY_1 + 2BY_2 + 2bY_1) $$

(Equation 6)

Equating $2R_{Sx}$ from Eq. 3 with $R_{bx}$ in Eq. 6, it can be proved that the optimum angle ($\beta_{opt}$) of inclination of the floor to achieve stable flow is given by

$$ \tan \beta_{opt} = \frac{2Y_1}{b} \tan \left[ \frac{Y_2 + Y_1}{2 + 2Y_1 + Y_2} \right] $$

(values of $\beta_{opt}$ obtained from Eq. 7) corresponding to different values of $2\Theta$ are given in line (ii) and Line (i) of Table 1 respectively.
3.2.2 Momentum and energy principles in expansion having reverse floor slope

Since the axial components of forces from side walls and bed are exactly balanced, the momentum and the energy equations may be expressed as

\[ F_b^2 = \frac{1}{2} \left( \frac{1 - y'^2}{1 - y''^2} \right) y'' \]  
\[ \frac{\Delta E}{Y_1} = (1 - y') - \frac{(1 + y')(1 - y''^2)}{4 y''} \cdot \Delta h \]  
\[ \Delta h = \frac{L_2 \tan \beta_{opt}}{Y_1} \]

where \( \Delta h = \frac{L_2 \tan \beta_{opt}}{Y_1} \). Theoretical values of \( \frac{\Delta E}{Y_1} \), as obtained from Eq. 8 and Eq. 9 are given by dotted lines in Fig. 4. For any given \( F_b \) and \( r \) values, the energy loss required \( (\frac{\Delta E}{Y_1}) \) in an expansion with reverse slope \( (\beta = \beta_{opt}) \) is much less than that required in the expansion with level bed \( (\beta = 0) \). If the actual head loss due to production of turbulence balances the required theoretical loss of energy in expansion with reverse bed slope, there will be negligible amount of residual K.E. resulting in uniform and stable flow downstream.

3.2.3 Experimental results

Experiments were conducted in expansion provided with reverse bed slope as stated under 3.2.1. Separation was completely eliminated and the flow downstream was found to be uniform and stable as shown in Fig. 7 & 8.

![Fig.7 separation free uniform & stable flow, \( \theta = 29^\circ \), \( \beta_{opt} = 6^\circ \)](image)

![Fig.8 Separation free uniform & stable flow, \( \theta = 60^\circ \), \( \beta_{opt} = 9.9^\circ \)](image)

\( \alpha_2 \)-values computed from measured distribution of velocity are given in line (iv) and the residual K.E. of flow in line (vi) of table-1.

4. CONCLUSIONS

Flow downstream of expansion with level bed is non-uniform. Higher is the angle of expansion \( (2 \theta) \), greater is the non-uniformity and instability of flow due to higher magnitudes of residual K.E. of flow moving downstream. Provision of optimum reverse bed slope \( (\beta_{opt}) \) remarkably improves the flow conditions downstream. The flow was almost uniform and stable right from the exist end of expansion, irrespective of the total angle of divergence \( (2 \theta) \). The residual K.E. of flow was negligible in all the cases. It is therefore, concluded from the above results that the non-uniformity and instability of flow arises due to residual K.E. of flow moving downstream of hydraulic structures.
TABLE I

<table>
<thead>
<tr>
<th>(i) Total angle of expansion (2 Θ)</th>
<th>22°</th>
<th>29°</th>
<th>52°</th>
<th>62°</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ii) Bed slope (βopt) for equality of axial reactions from bed &amp; side walls (from eq. 7)</td>
<td>5.1°</td>
<td>6.3°</td>
<td>8.8°</td>
<td>9.9°</td>
</tr>
<tr>
<td>(iii) α2 (with level bed)</td>
<td>1.65</td>
<td>3.00</td>
<td>4.87</td>
<td>6.40</td>
</tr>
<tr>
<td>(iv) α2 (with sloping bed)</td>
<td>1.12</td>
<td>1.14</td>
<td>1.18</td>
<td>1.17</td>
</tr>
<tr>
<td>(v) Residual K.E. of flow (Level bed) ( i.e. (ΔE - ΔE')/ν_k ) for ( β = 0° )</td>
<td>0.0187</td>
<td>0.0420</td>
<td>0.0435</td>
<td>0.0460</td>
</tr>
<tr>
<td>(vi) Residual K.E. of flow (sloping bed) ( i.e. (ΔE - ΔE')/ν_k ) for ( β = β_{opt} ) in (ii)</td>
<td>0.0070</td>
<td>0.0070</td>
<td>0.0070</td>
<td>0.0053</td>
</tr>
<tr>
<td>(vii) Residual K.E. ratio between level and sloping bed (v/vi)</td>
<td>2.67</td>
<td>6.00</td>
<td>6.21</td>
<td>8.67</td>
</tr>
</tbody>
</table>

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