CONTROL OF SEPARATION IN OPEN CHANNEL SUB-CRITICAL EXPANSION WITH ADVERSE BED SLOPE

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ABSTRACT

With gradual increase in the angle of divergence of the side walls, the different regimes of flow which appear successively in a sub-critical expansion with level bed have been described. The critical angles of divergence corresponding to the different flow regimes have been found experimentally and plotted against the geometry of the expansive passage. Separation of flow and eddies were eliminated by providing adverse slope to the expansion floor. Performance of the expansions with level and sloping beds have been compared to demonstrate the effectiveness of the technique to control separation in sub-critical flow expansion.

INTRODUCTION

Keeping the length of wall(L) constant, if the total angle of divergence (2θ) is gradually increased, the various flow regimes that appear successively in a sub-critical flow expansion are shown in fig.1. At a very small angle, the flow is free from any separation (Fig.1a). Beyond a critical angle (2θ1), intermittent eddies are observed (fig.1b). A fully developed eddy (fig.1c) appears at a critical angle (2θ2). 2θ3 is the critical angle at which eddies appear on both the sides and a jet flows through the center (fig.1d).

![Fig.1 Different regimes of flow](image-url)

SEPARATION CONTROL BY ADVERSE BOTTOM SLOPE

According to fig. 2, the flow through an expansion is subjected to an axial force component $R_{sx}$ from the side walls. Assuming

\[ 2R_{sx} = \frac{1}{3} \rho g L_a \tan \theta (y_1^2 + y_2^2 + y_1 y_2) \]  
(1)

and

\[ R_{bx} = \frac{1}{3} \rho g L_a \tan \beta (by_2 + By_1 + 2By_2 + 2by_1) \]  
(2)

Where $\rho$ is the density of water, $g$ is the acceleration due to gravity, $y_1$ and $y_2$ are the depths, $b$ and $B$ are the widths upstream and downstream of the expansion respectively. Equating (1) and (2) and simplifying, the optimum adverse slope ($\beta_{opt}$)
of the expansion floor may be determined from equation-3.

\[ \beta_{\text{opt}} = \tan^{-1}\left[ \frac{2y_1/b}{\tan \theta (1+y'+y'^2)/(2+2y'r+y'+r)} \right] \]

(3)

Where \( y' = y_2/y_1 \), \( r = B/b \). For a given discharge \( (Q=10 \text{Lps}) \), a given Froude's number of flow at entry \( (F_b = 0.6) \) and \( b = 15 \text{ cm} \), values of \( \beta_{\text{opt}} \) are 5.1°, 6.3°, 8.8° and 9.9° corresponding to \( \theta \)-values of 22°, 29°, 52° and 62° respectively.

**PERFORMANCE CRITERIA**

Efficiency \( (\eta) \) of an expansion may be defined as

\[ \eta = \frac{\Delta y}{(v_1^2/2g - v_2^2/2g)} \]

(4)

Where \( \Delta y = \) recovery of head \( (\text{fig.2}) \), \( v_1 \) \& \( v_2 \) are the mean velocity of flow before and after the expansions. Another important criteria is the uniformity of velocity distribution just at the exit of the expansion as indicated by Coriolis's coefficient

\[ \alpha_2 = \frac{1}{A_2} \int v_2^3 \, dA \]

(5)

Where \( A_2 \) = area of flow section at the exit, \( u \) is the local velocity of flow through an elementary area \( dA \). Higher is the \( \eta \)-value, lower will be the value of \( \alpha_2 \), since very little residual kinetic energy will move downstream \( (\text{fig.2b}) \). In an efficient expansion (with high \( \eta \) and low \( \alpha_2 \)), there will be hardly any separation at the exit of expansion.

**PERFORMANCE OF EXPANSION WITH LEVEL \((\beta = 0^\circ)\) AND SLOPING \((\beta = \beta_{\text{opt}})\) BEDS**

Fig.3 gives the critical angles \( \theta_1, \theta_2 \) and \( \theta_3 \) corresponding to the different flow regimes obtained with level bed \( (\beta = 0^\circ) \). Low \( \eta \)-values and high \( \alpha_2 \)-values \( (\text{Table-1}) \) indicate extremely poor performance of the level bed expansion at all the angles \( 20=22^\circ, 29^\circ, 52^\circ \) and \( 62^\circ \). Separation was completely eliminated by use of adverse slope \( (\beta = \beta_{\text{opt}}) \) to the expansion floor as shown in fig.4(a) & (b) and fig.5(a) & 5(b). Comparing the \( \eta \) and \( \alpha_2 \)-values \( (\text{Table-1}) \) obtained with level and sloping beds, it is apparent that the performance of an expansion with adverse bed slope \( (\beta_{\text{opt}}) \) is far superior.

![Fig.3 Critical angles for different flow regimes](image-url)
Table 1 Comparison of Performance of Expansion with and without Bed Slope

<table>
<thead>
<tr>
<th>Total angle of expansion (θ)</th>
<th>Performance of Expansion</th>
<th>With Level Bed (β = 0°)</th>
<th>With Adverse Bed Slope (β_{opt})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β^0</td>
<td>η</td>
<td>α_2</td>
</tr>
<tr>
<td>22°</td>
<td>0</td>
<td>17</td>
<td>1.65</td>
</tr>
<tr>
<td>29°</td>
<td>0</td>
<td>13</td>
<td>3.00</td>
</tr>
<tr>
<td>52°</td>
<td>0</td>
<td>11</td>
<td>4.87</td>
</tr>
<tr>
<td>62°</td>
<td>0</td>
<td>10</td>
<td>6.4</td>
</tr>
</tbody>
</table>

to that obtained in case of level bed, for all the angles of expansion.

CONCLUSIONS

(i) Various flow regimes in open channel sub-critical flow expansion with level bed are found to be governed by the length (L/b) and total angle of expansion (θ).
(ii) An effective method of separation control in an expansion is to provide adverse slope (β_{opt}) to the floor, such that the bed reaction (R_{bx}) exactly balances the wall reactions (2R_{sx}).
(iii) Hydraulic performance of expansion with optimum adverse bed slope (β_{opt}) given by eq.(3) is excellent when compared with the one having level bed (β=0°).

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Fig. 4 Flow Pattern (a) with fully developed eddy on one side at $\theta = 29^\circ$, $\beta = 0^\circ$, (b) free from any separation at $\theta = 29^\circ$, $\beta_{opt} = 6.3^\circ$
Fig. 5 Flow Pattern showing (a) Jet flow with eddies on either side ($2\theta=62^\circ, \beta=0^\circ$), (b) free from separation at ($2\theta=62^\circ, \beta=9.9^\circ$)
REFERENCES


LIST OF SYMBOLS

\[\begin{align*}
A_2 & \quad \text{Cross-sectional area of flow at exit of expansion} \\
b & \quad \text{Width at the throat} \\
P_b & \quad \text{Froude's number of flow at entry to the expansion} \\
R_{bx} & \quad \text{Axial component of bed reaction} \\
L & \quad \text{Length of expansion wall} \\
L_a & \quad \text{Axial length of expansion wall} \\
Q & \quad \text{Flow in L/sec} \\
r & \quad \text{B/b} \\
R_{sx} & \quad \text{Axial component of wall reaction} \\
V_2 & \quad \text{Mean velocity of flow at exit} \\
\rho & \quad \text{Unit weight of water} \\
\gamma_1 & \quad \text{Depth of flow at entry to expansion} \\
\gamma_2 & \quad \text{Depth of flow at exit end of expansion} \\
\alpha_2 & \quad \text{Corioli's coefficient at exit end of expansion} \\
\beta & \quad \text{Slope of bed of expansion with horizontal} \\
\beta_{opt} & \quad \text{Optimum bed slope of expansion} \\
\Delta y & \quad \text{Recovery of head} = (\gamma_2-\gamma_1) \\
2\theta & \quad \text{Total angle of expansion} \\
\eta & \quad \text{Efficiency of expansion}
\end{align*}\]