LIMITATIONS OF LACEY’S THEORY FOR ESTIMATION OF BRIDGE SCOUR

S.K. Mazumder, Advisers
ICT Pvt. Ltd., A-8 Green Park, New Delhi-14 and
(Former AICTE Emeritus Professor of Civil Engg., DCE)
E-mail Address: somendra64@yahoo.com

ABSTRACT
Estimation of scour is necessary for economic and safe design of bridges. Developed countries are using mathematical models after validating them by on-site measurement of scour at bridge sites. Indian codes for scour estimation are based on Lacey’s regime equations several limitations of which are pointed out. Maximum scour depths in bridge piers founded on non-cohesive alluvial soil are computed in five major bridges in India by using four mathematical models and compared with those obtained from Indian codes. It is noticed that in all the cases Lacey’s model overestimates scour depth with errors varying from 2% to 275%.

INTRODUCTION
Estimation of scour in bridges is extremely important for the design of bridge foundation, guide bunds, protection works etc. Any under estimation of scour will result in failure of the bridge whereas over-estimation of scour will lead to escalation of its cost. In major bridges, cost of foundation, guide bunds and protective works exceed 50% of the total cost of a bridge.

In India, scour estimation is done by use of codes/guidelines published by Bureau of Indian Standards (IS: 6966, IS: 10751) Indian Roads Congress (IRC-5, IRC-78) for highways and RDSO in railways Procedure recommended for scour estimation in all these codes are based on theory propounded by Lacey (1930) and Blench (1969) which are empirical in nature and developed at a time when there was very little knowledge about mechanism of sediment transport and scouring processes. In the developed countries, however, estimation of bridge scour is made by using scientific mathematical models by eminent river engineers e.g. Laursen (1981), Melville & Coleman (2000), Breussers and Raudkivi (1991), Richardson and Davis (1995) – all of which have been validated by measurement of scour at bridge sites by use of sophisticated instrumentations. (USGS-1999, EWRI-2007). In India too, extensive research on bridge scour has been performed by Kothyari, Garde & RangaRaju (1992a, 1992b), Gangadharaiah (1985), Dey (1995). Unfortunately, none of the mathematical models either from India or from abroad have been validated by measuring prototype scour at bridge sites under Indian river conditions. There is hardly any reliable scour data from bridge sites. The Indian codes are, therefore, still following Lacey’s regime equations for estimation of scour in bridge design.

Lacey’s regime equations were developed in India on the basis of extensive data collected from stable canals in the Bari Doab areas in west Punjab (now in Pakistan). But they have several limitations. Scour estimated by indiscriminate use of Lacey’s regime equations, irrespective of river behavior (Mazumder, 2004), river plan and bed forms (Garde, 2000), properties of river bed materials, geometry of the bridge piers and abutments and the approach channel geometry etc. (Melvle and Coleman, 2000), may substantially differ from actual one. In a technical paper presented by the author (Mazumder, 2006) at the 67th Indian Roads Congress, scour computed by Lacey’s method in piers in five major bridges in India are compared with those computed by four different mathematical models. It is found that in all the cases Lacey’s model overestimates scour and the error varies from 2% to 275%,
One of the primary objectives of writing this paper is to discuss the limitations of Lacey’s model in scour estimation in bridges and the necessity of updating the Indian codes for scour estimation by introducing rational and scientific mathematical model. It is necessary to collect scour and other parameters by deployment of suitable measuring instruments for reliable measurement of scour and other relevant data governing scour at bridge sites under Indian river conditions for the validation of mathematical models and updating Indian codes. An MoU has recently been signed between IIT (Roorkee) and ICT (New Delhi) for a joint collaborative research to meet the above objectives. The research proposal is submitted to MOSRT&H, Govt. of India, for consideration of research funding.

**IRC/IS/RDSO METHOD FOR ESTIMATING MAXIMUM SCOUR DEPTH**

IRC/IS/RDSO codes recommend that the maximum scour depth ($Y_{\text{max}}$) is to be found simply by multiplying regime depth ($R$) obtained by Lacey’s/Bleich’s equations (given by equations 1 to 4 below) with a factor $K$-varying from 1.27 to 3 depending on scour location (CBI & P, 1989). The codes recommend that $R$ should be found for a higher discharge intensity for determining scour depths in bridge piers and abutments by increasing the design discharge ($Q_{50}$) by an amount varying from 10% to 30% of the design discharge ($Q_{50}$) depending upon the area of catchments, where ($Q_{50}$) stands for a flood discharge of 50 year return period. With the above equations, the maximum scour depth for piers are computed for five major bridges in India and are given in table-1 (a) & 1 (b).

**MATHEMATICAL MODELS FOR ESTIMATION OF SCOUR DEPTH IN BRIDGES**

In all mathematical models, total scour depth in bridge piers and abutments is found by adding up (i) general scour (ii) constriction/contraction scour and (iii) local scour found separately. Some of the popular mathematical models used for estimation of scour in bridges are Melville & Coleman (2000) model in Newzealand, HEC-18 (Richardson and Davis,1995) model in USA, Kothyari-Garde-RangaRaju (1992) model in India and IAHR (Breussers and Raudkivi ,1991) model in Europe. The above four mathematical models used for estimating maximum total scour depth in five major bridges in India are briefly discussed in the following paragraphs.

**General Scour**

General scour is the scour which occurs irrespective of the presence of the bridge due to the various morphological processes in a river, namely, aggradation and degradation of river bed, meandering, braiding, cut-off formation, confluence of streams upstream of bridge sites, etc. Lacey (1930), Blench (1969), Neill (1973), Chitale (1981),Yalin (1999), Diplas (1990), Garde and Rangaraju (2000), and many eminent river engineers have done commendable works to find the dimensions of a stable channel section or a regime section. The most popular method of predicting general scour by using Lacey’s and Blench’s regime equations (used by author) is discussed below.

**(a) Lacey’s Model**

In India, general scour depth in bridge piers, abutments, guide bunds, protective works etc. is computed by use of Lacey’s model. Lacey’s regime depth ($R$) measured below high flood level for determining foundation level and regime waterway for deciding bridge span are given by the empirical equations -1, 2 &3

\[
R = 0.473 \left( \frac{Q}{f} \right)^{1/3} \\
P = 4.75 Q^{1/2} \\
f = 1.76 \left( d_{50} \right)^{1/2}
\]

Where $R$ and $P$ are regime depth and waterway respectively in meter, $Q$ is the design
discharge in cumec, \( f \) is Lacey’s silt factor and \( d_{50} \) is the mean weighted size of non-cohesive sediments in the river bed and bank in mm.

**b) Blench’s Model**

Where the normal/Lacey’s waterway of the stream is restricted (Mazumder, 2002), Indian codes recommend that \( R \) should be determined from Eq. 4 proposed by Blench (1957).

\[
R = 1.34 \left( \frac{q^2}{f} \right)^{1/3}
\]

where \( q \) is the intensity of discharge under the bridge given by the relation

\[
q = \frac{Q}{W} = V Y
\]

where \( W \) is the effective clear waterway, \( V \) is the mean velocity of flow and \( Y \) is the flow depth under the bridge in meter units. \( R \) in Lacey/ Blench equations give the general scour depth in a straight channel. In curved channels, as in river bends, general scour depth will be more due to secondary currents and erosion on outer banks (Neil, 1973).

General scour depths (measured below HFL) estimated by equation -4 (since all the five bridges have restricted waterway) are given in Table-1.

**Constriction/ Contraction Scour**

Constriction or contraction scour occurs in a bridge where the road or railway approach embankment restricts the normal waterway. Lowering of the bed occurs locally within the contracted reach (i.e. under the bridge) due to flow acceleration and increased velocity of flow. Estimation of constriction scour should be done depending on whether the bed is stable (rigid) or live (mobile). The bed becomes mobile when the mean velocity of flow (\( V \)) in the channel exceeds the critical velocity (\( V_c \)) or the bed shear stress (\( \tau_o \)) exceeds the critical shear stress (\( \tau_c \)) at which the stream bed material just starts moving.

Laursen (1981) have contributed immensely for finding scour and flow characteristics due to restriction of waterway in a bridge. For the case of clear water scour, \( (\tau_o < \tau_c \) or \( V < V_c) \), Richardson and Davis (1995) recommend the following equation for computing scour depth in a constriction.

\[
Y_2 = 1.48 \left( \frac{Q_2}{(d_{m1/3} W_2)^{6/7}} \right)
\]

Where \( Y_2 \) is the average depth including scour depth under the bridge in meter, \( Q_2 \) is the total discharge through bridge in cumec, \( d_{m} \) is effective mean diameter of the bed material in mm (\( d_{m} = 1.25 d_{50} \)), \( W_2 \) is the average bottom width of river under the bridge in m and the constant (1.48) has a dimension (L^{-3/7}). It is assumed that the scour continues to occur in the contracted reach until threshold condition is attained. Constriction scour depth (\( d_{sc} \)) measured below original river bed is given by

\[
d_{sc} = (Y_2 - Y_1),
\]

where \( d_{sc} \) is the scour depth in m below bed and \( Y_1 \) is the original depth of flow in m at the contracted site before the construction of the bridge.

Live bed scour (\( \tau_o > \tau_c \) or \( V > V_c \)) at a contracted section can be found by the equation proposed by Richardson and Davis (1995) as follows:

\[
Y_2/Y_1 = (Q_2/Q_{1m})^{6/7} \ (W_1/W_2)^{K_1}
\]

Where \( Q_{1m} \) is the discharge in the approach channel transporting bed sediments, \( Q_2 \) is the total discharge passing under the bridge, \( K_1 \) is a coefficient varying from 0.59 (for sediments transported mostly as bed load) to 0.69 (for sediment transport mostly in suspended form), \( W_1 \) and \( W_2 \) are the mean widths of the stream in the approach channel and the contracted section under the bridge respectively.

Constriction scour depths, measured below lowest river bed, in all the five rivers found by using equation- 6 (since the river beds in all the five bridges are in live conditions) are given in table-1.
Local Scour

A large numbers of mathematical models have been developed over the years in India and abroad for estimating local scour depths around bridge piers and abutments due to the flow obstruction and formation of vortices. Four such mathematical models are used in estimating local scour depths in bridge piers in five major Indian bridges founded on alluvial non-cohesive soil (see table-1). These are given by equations 7 to 12 below

(a) Melville and Coleman Model (2000)

\[ d_{se} = K_{yb} \cdot K_{i} \cdot K_{a} \cdot K_{al} \cdot K_{g} \cdot K_{t} \]  

(b) HEC-18 Model (After Richardson and Davis, 1995)

\[ \frac{d_{se}}{y_1} = 2K_{s} \cdot K_{al} \cdot K_{3} \cdot \frac{b}{y_1}^{0.65} \cdot F_{r1}^{0.43} \]  

(c) Kothyari – Garde - RangaRaju Model (1992)

For clear water scour depth \( (d_{se}) \) measured below bed:

\[ \frac{d_{se}}{d_{50}} = 0.66 \left( \frac{b}{d_{50}} \right)^{0.75} \left( \frac{y_1}{d_{50}} \right)^{0.16} \left( \frac{V^2 - V_c^2}{\rho \Delta \gamma_s} \right) \alpha^{-0.30} \]  

For live or mobile bed scour depth measured below bed:

\[ \frac{d_{se}}{d_{50}} = 0.88 \left( \frac{b}{d_{50}} \right)^{0.67} \left( \frac{y_1}{d_{50}} \right)^{0.4} \alpha^{-0.3} \]

(d) IAHR Model (After Breussers & Raudkivi, 1991)

For clear water local scour \( (d_{se}) \) when \( u < u_{se} \), or \( V < V_c \)

\[ \frac{d_{se}}{b} = 2.3K_{a}K_{(b/d_{50})}K_{b/d_{50}} \]  

for live bed scour when \( u > u_{se} \), or \( V>V_c \), the equation is

\[ \frac{d_{se}}{b} = X \cdot K_{(b/d_{50})} \cdot K_{b/d_{50}} \cdot K_{t} \]  

where \( d_{se} \) is equilibrium local scour depth (see fig.1), \( b \) is the thickness of pier, \( y_1 \) is the u/s flow depth, \( K_{yb} \) is depth size factor having same dimension as \( d_{se} \) and dependent on \( b/y_1 \) ratio; all other K- values are non- dimensional, \( K_{al} \) and \( K_{a} \) are pier alignment and shape factors respectively, \( K_{g} \) is sediment non-uniformity factor, \( K_{i} \) is flow intensity factor, \( K_{b/d_{50}} \) is sediment- pier size factor, \( K_{3} \) is a correction factor for bed condition i.e. plain bed, ripple and dune bed etc., \( K_{4} \) is the correction factor due to armoring of bed in non-uniform sediments, \( K_{d} \) is sediment size factor, \( K_{r} \) is time factor (with respect to time to attain equilibrium scour \( d_{se} \), \( F_{r1} \) is approach flow Froude’s number, \( \alpha = (B-b)/B \), \( B \) is the distance between two consecutive piers, \( \Delta \gamma_s = \gamma_s - \gamma_f \), \( \gamma_s \& \gamma_f \) are the unit weights of sediment and water respectively;\( \bar{V} \) and \( V_c \) are the mean velocity of flow and critical flow velocity at threshold condition respectively, value of X is 2.3 when \( V > 4V_c \); When \( V_c < V < 4V_c \), value of X varies from 2 to 2.30 for uniform sediments (\( \sigma_g \leq 1.3 \)) and “X” varies from 0.5 to 2.0 for non-uniform sediments (\( \sigma_g > 1.3 \)). The values of the above parameters are given in the references cited against the models.

Local scour depths found from equations 7 to 12 and the total scour depth found by adding up general scour, constriction scour and local scour depths for the five major bridges and are given in Table-1(a) and 1(b).
COMPARISON OF SCOUR DEPTHS OBTAINED BY DIFFERENT METHODS

Table 1(a) gives the maximum scour depths computed by different methods, assuming that the bed profile of the rivers develop to Lacey’s/ Blench’s type profile during the passage of high flood. Table 1(b) gives the maximum total scour depth assuming that the lowest bed level remains unchanged. It is noticed from the tables that IRC/IS/RDSO methods overestimates scour in all the cases and the percentage errors (with respect to total scour obtained by different mathematical models and given in brackets under the column total scour depth in the respective models) is found to vary from 2% to 275%.

LIMITAIONS OF LACEY’S MODEL

Some of the limitations of Lacey’s model adopted by IRC/IS/RDSO for finding scour depth in bridge piers and abutments are stated below.

- It does not distinguish between general, contraction and local scour.
- It finds MSL with respect to HFL resulting in level scoured surface.
- It does not consider many of the important parameters e.g. flow shallowness; geometry, length & alignment of piers and abutments; incoming sediment load and debris; size, shape and non-uniformity of sediments etc.
- It is independent of actual time of scouring with respect to time required to attain equilibrium scour (fig.1).
- Lacey’s R-value is applicable for steady flow in fine incoherent alluvial soils only
- Lacey’s silt factor \( f = 1.76 \sqrt{d_{50}} \) depends on many other parameters besides mean size of sediments- \( d_{50} \).
- It neglects the morphology of a river- its various plan and bed forms.
- Lacey’s theory is applicable for canal flow which is steady, continuous and for a prolonged period. It can at best be used for finding approximate general scour depth but definitely not for localized scour depth around bridge piers and abutments governed by several other parameters.
- Sediment transport principles and scouring processes under live bed and clear water conditions are totally ignored in Lacey’s method of scour estimation. It is well established universally that scour is maximum at threshold/critical/incipient condition of bed motion when \( Q=Q_c \) or \( V=V_c \) or \( \tau=\tau_c \) at \( t=t_c \) (where \( Q, V, \tau, t \) are discharge, velocity, shear stress and time respectively and the critical values of the same are denoted with subscript \( c \)). Scour reduces thereafter with further rise in the value \( Q, V & \tau \) i.e. when \( Q>Q_c, V>V_c & \tau>\tau_c \) and attains an equilibrium state at about \( V=4V_c \) as shown in Fig.1. In Lacey’s method, however, scour is independent of the critical conditions and it goes on increasing with increasing values of \( Q, V, \tau \) and it is independent of time which is far from truth.

![Fig. 1 Variation in Scour Depth (ds) with Mean Flow Velocity (V) showing Clear Water, Threshold and Live Bed Scour in Bridge Piers & Abutments (Mellvile, 2000)](image-url)
Table - 1 Scour Depths Computed By Different Methods

(a) Assuming that the low water bed profile develops to Lacey's regime profile in flood

(b) Assuming that the low water bed profile remains uncharged during flood

<table>
<thead>
<tr>
<th>Name of River Crossing (NH No.)</th>
<th>General scour depth below HFL (As per Regime theory)</th>
<th>Constriiction scour depth below mean bed level</th>
<th>Total scour depth by Lacey (IRC method)</th>
<th>Local scour below bed and total scour below HFL (i.e., sum total of general scour, consriction scour and local scour)</th>
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<td>Melville &amp; Coleman</td>
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<td>Richardson &amp; Davis (HEC-18)</td>
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<td>Breussers &amp; Raudkivi (IAHR)</td>
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<td>Kothyari, Garde &amp; Ranga Raju</td>
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<td>Chambal (NH-3)</td>
<td>23.80</td>
<td>6.83</td>
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<td>37.83 (22.3%)</td>
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<td>43.81 (5.6%)</td>
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<td>Saryu (NH-28)</td>
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<td>Raidak-I (NH-31C)</td>
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<td>Sankosh (NH1-C)</td>
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<td>11.74 (16.7%)</td>
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NOTES: (i) All scour depths are in meter. Total scour depth is subtracted from Design HFL to obtain maximum scourlevel (MSL) around the pier. Constriction and local scour depths are below mean bed level of the stream (ii) Figures in brackets indicate percentage excess total scour by Lacey's (IRC) method with respect to other methods. (iii) General scour depth of flow is computed as the average of regime flow depths computed by Lacey's and Blench's theories.
CONCLUSION
The current method of scour estimation as prescribed in IRC, RDSO and IS Codes (used in India) is based on Lacey’s regime theory developed in 1930. Lacey’s method has several limitations as it ignores many important parameters, namely, geometric, hydrologic, hydraulic, flow, sediment transport, properties of foundation materials etc. Several mathematical models have been developed over the years for precise estimation of general scour, contraction scour and local scour. Scour around piers in 5 bridges are computed using four different mathematical models of Melville & Coleman, Richardson & Davis (HEC-18), Breussers & Raudkivi (IAHR), Kothyari-Garde and Rangaraju. Total depths of scour found from first four models have been compared with scour depths found by Lacey’s method adopted by Indian codes. Percentage excess scour found by Lacey’s method with respect to other mathematical models range between 2.4% to 90% in table-1 (a) where general scour depth is taken as regime depth of flow. The corresponding figures are found to vary between 10.2% and 275.2% in table-1(b) where the general scour is taken as the mean flow depth measured from HFL to mean bed level (observed during low flow). It is, however, difficult to conclude which mathematical model gives the best result unless the results are compared with actual scour measurement in prototype at different bridge sites.

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