DESIGN OF CONTRACTING AND EXPANDING TRANSITIONS IN OPEN-CHANNEL SUB-CRITICAL FLOW

by

S.K. Mazumder

Assistant Professor, Regional Engineering College
Durgapur (India)

SYNOPSIS

Different types of open-channel transitions with various kinds of flow regimes have been described, pointing out the scope of the present design. Characteristics of sub-critical flow in the contracting and expanding zones of the transition have been discussed. Performances required to be fulfilled by the transitions have been stated in terms of: (*i*) hydraulic efficiency, (*ii*) bed shear distribution, and (*iii*) separation pattern. Review of literature on the subject is made and the inefficiency of the existing design has been focussed. A new trial and error method for the design of contracting transition based on linear change in mean velocity has been introduced. Limitations of the conventional design of expanding transition have been critically examined. Based on principles of boundary layer flow control, attempt has been made to design an efficient wide-angle expansion by providing a pair of triangular submerged vanes, symmetrically placed converging downstream in plan at the commencement of expansion. Design curves for the optimum dimensions of the vanes, its spacing, orientation, etc., have been prepared from the graphical plot of the several performances for different flows. The performances of the expansion provided with vanes have been compared with those for other conventional designs without having any appurtenances. An example has been worked out in the appendix to illustrate the design procedure of contracting and expanding transition.

RESUME

On décrit plusieurs types de raccordements des canaux découverts avec divers types de régimes d'écoulement, en signalant la portée de la présente étude. Les caractéristiques de l'écoulement sous-critique dans les zones contractantes et dilatantes des raccordements ont été examinées. Les rôles de ces raccordements sont définis en fonction de (1) l'efficacité hydraulique, (2) de la distribution du cisaillement de lit, et (3) du type de séparation. On parle de la revue de la littérature sur le sujet et on attire l'attention sur l'étude actuelle. On a introduit une nouvelle méthode d'essai par tâtonnements pour l'étude du raccordement contractant à la base du changement linéaire en vitesse moyenne. On a examiné critiquement les limitations de

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* Etude des transitions contractantes et dilatantes dans l'écoulement sous-critique du canal.

l'étude classique du raccordement dilatant. A la base des principes de régulation du débit de la couche limite, on a tenté de projeter une expansion efficace de grand angle en prévoyant une paire de parois triangulaires noyées installées symétriquement vers l'aval au début de l'expansion. Les courbes de dessin pour les dimensions optimales des parois, leur espacement, orientation, etc ont été tracées à partir de la parcelle graphique de plusieurs performances pour divers écoulements. Les performances de l'expansion prévue avec les parois ont été comparées avec celles destinées aux autres dessins classiques dépourvus de tout accessoire. On a cité un exemple dans l'appendice en vue d'illustrer la procédure de dessin pour les raccordements contractants et dilatants.

1. INTRODUCTION

1.1. In open channels, there are several situations when the normal section of flow is constricted for reasons of economy. In falls, aqueducts, siphons, bridges, flumes and in many other similar hydraulic structures, the original cross section of flow is reduced so as to economise the construction costs. As another example, fluming of channel offers an expedient device for measurement of discharge, e.g., in Parshall flumes, critical flow meters and Venturi flumes.

1.2. All transitions may be classified as either contracting or expanding. A contracting transition connects the normal section to the constricted section (often termed as throat) and the expanding transition joins the constricted section to the normal section again, moving in the direction of flow.

1.3. Transition may be brought about in any of the following ways:

(*a*) Change in width, bed remaining level.

(*b*) Change of elevation of bed in vertical plane, width remaining constant.

(*c*) Change in both width and bed level simultaneously.

The scope of the present paper is limited to Case (*a*) only.

2. VARIOUS REGIMES OF FLOW

2.1. A transition may be designed with a view to bringing in any of the following changes in flow regime, namely:

(*a*) Sub-critical to sub-critical flow.

(*b*) Super-critical to super-critical flow.

(*c*) Sub-critical to super-critical flow and vice-versa.

2.2. The design prescribed in this paper is meant only for Case (*a*). Under Case (*a*) may again be included three different classes of transitions. They are:

Class I: Transition in which velocity never rises above critical.

Class II: Transition in which the velocity rises above the critical but the design is such that the stream returns again to sub-critical stage at the exit without any standing wave.

Class III: Transition in which the velocity rises above critical and in which provision is made for generation of standing wave at a point where the stream resumes sub-critical velocity.

The transition studied herein belongs to Class I.

3. CHARACTERISTICS OF FLOW THROUGH TRANSITION

3.1. Flow through a transition is non-uniform. Depending on the degree of constriction and the rate of flaring, the flow may either be rapidly varied or gradually varied in nature. Due to curved surface profile pressure distribution is not hydrostatic. The resistance offered by the bed and sides renders the velocity distribution non-uniform in any given cross sections, resulting in values of momentum and energy correction factors far greater than unity. In the contracting zone, the pressure gradient is negative and in the expanding portion, the gradient is positive, as the flow is sub-critical. It has been proved both theoretically and experimentally that the growth of boundary layer is intimately related with external pressure gradient. In a contracting transition, the pressure gradient being negative, the thickness of the boundary layer reduces in the direction of flow; whereas in an expanding transition, associated with positive pressure gradient the thickness of boundary layer increases in the direction of flow. Thus, the flow in a contracting transition is stable, as the velocity distribution is more uniform (thin boundary layer) and the favourable pressure gradient helps the slowly moving boundary layer particles to overcome friction. In the expanding reach, however, the flow is mostly unstable, as the velocity distribution is more non-uniform (thick boundary layer) and the adverse pressure gradient acts along with friction in resisting the boundary layer particles. As the result, the flow often separates from the boundary, leading to generation of turbulence and eddies. Substantial portion of the kinetic energy, which could have been transformed to pressure head, gets eaten away through production of turbulence which is converted, diffused and dissipated by the main stream subsequently. Such eddies may also be formed in contractions, too abrupt. The adverse pressure gradient due to stagnation at sides causes the boundary layer to separate and eddies are generated at the sides.

4. PERFORMANCE OF TRANSITION

4.1. In open-channel, transitions are provided to perform any one or more of the following functions :

(*a*) To minimise head loss so that the afflux is minimum. Too much afflux affects regime condition and needs extra heightening of banks upstream. Increased afflux may submerge the head regulator of a canal and thereby affect supply to it.

(*b*) To prevent erosion of bed and sides upstream and downstream of transition. If the design of transition is not proper, the flow may separate from the boundary, leading to whirlpool eddies and extremely non-uniform distribution of velocity. Such eddies as well as local concentration of shear resulting from non-uniform distribution in velocity, result in huge scour of the channel both at bed and sides, threatening thereby the very existence of the structure for which the transition is provided.

(c) To achieve a smooth, separation free, stable flow, free from any cross waves and undesirable stalls. Eddies are the sources of energy loss, create pressure fluctuations and may lead to objectionable vibration of structures.

4.2. The design outlined herein aims at fulfilling all the three objectives stated above.

5. REVIEW OF LITERATURE

5.1. In open channel, various forms of transitions having different lengths (governed by average side splay varying from 1 : 1 to 10 : 1 or so) had been constructed. Very little was known about the field performance of these transitions. The first major investigation in this respect was undertaken by Julian Hinds(5). From analysis of field data, Hinds observed that those transitions which were gradual and smooth gave better performance, in comparison with those having abrupt discontinuity in boundary and less gradual. In his paper 'The Hydraulic Design of Flume and Siphon Transitions', Hinds presented a number of empirical rules governing the design of transition structures.

5.2. United States Bureau of Reclamation(1) (U.S.B.R.) developed a variety of transitions for open channel sub-critical flow. Different U.S.B.R. transitions with corresponding head loss coefficients $C_{J(inl)}$ and $C_{J(out)}$ " for inlet and outlet transitions respectively, are given below :

Type of Transition	$C_{J(inl)}$	$C_{J(out)}$
Square ended type	0.30	0.75
Straight line type	0.30	0.50
Wedge type	0.20	0.30
Cylinder quadrant type	0.15	0.25
Warped type	0.10	0.20

where,

$$C_{J(inl)} = \frac{J_{(inl)}}{\left(\bar{v}^2_{(t)}/2g - \bar{v}^2_1/2g\right)} \quad \text{and} \quad C_{J(out)} = \frac{J_{(out)}}{\left(\bar{v}^2_{(t)}/2g - \bar{v}^2_2/2g\right)}$$

$\bar{v}_{(t)}$, $\bar{v}_1$ and $\bar{v}_2$ stand for the mean velocities of flow at the throat, upstream and downstream sections respectively. $J_{(inl)}$ and $J_{(out)}$ are the losses in head at inlet and outlet transitions respectively. U.S.B.R. recommends simple forms, *e.g.*, cylinder quadrant or straight type for unimportant structures, where velocity is small and head loss is not of much moment. But in important structures, *e.g.*, aqueducts, siphons, where fluming is severe and head loss should be minimised, U.S.B.R. recommends warped type transition popularly known as Hinds' transition. Design of warped type transition is based on several assumptions, the most important of them being as follows :

(*i*) Water surface profile is a compound curve made up of two reverse parabolas, with the inflection point at the middle and merging into the upstream and downstream water surface at either end of the transition tangentially.

(*ii*) Loss coefficients $C_{J(inl)}$ and $C_{J(out)}$ remain constant throughout the length of transition.

(*iii*) Length of transition is given by an angle 12°-30′ between the axis of the channel and the line joining the points where the water surface meets the sides at entry and exit of the transition. Procedure of design is tedious and time consuming. Assumptions made may be far from actual state of affairs. $C_{J(inl)}$ and $C_{J(out)}$ values are actually found to vary with flow characteristics, *e.g.*, depth, and discharge and boundary characteristics, *i.e.*, expansion ratio, surface conditions. The length of transition as arbitrarily assumed by an angle 12°-30′ may also be subject to criticism. As regards shape of water surface, Hinds' own conclusion was "no definite data as to the best form of water surface profile, the best form of structure of the most efficient length of transition are available." Nor do we know anything positive about the interrelationship of length, the water surface profile and the shape of transition and for that matter degree of flaring suitable under any particular boundary and flow conditions.

5.3. Amongst several authors who have studied the performance of transitions of various forms derived either empirically or semi-empirically, mention may be made of Mitra(12), Chaturvedi(8), Formica(3), Smith and Yu(16). The Author, in his Master's thesis(7), adopted transition representing the boundary of an eddy zone occurring in an abrupt contraction or abrupt expansion. The shape of the curve, found experimentally by Isbash and Lebedev(2) is given by :

$$Y = \tfrac{1}{2}B_{(t)} + \tfrac{1}{2}\,(B - B_{(t)}) \cdot \frac{X}{L_{(trans)}}\,\left(1 - \sqrt{1 - X/L_{(trans)}}\ \right)$$

where, X and Y are the co-ordinates of any point on the curve measured from origins which lie at the junctions of the centre line and the inlet and outlet ends of the throat, for the contracting and expanding transitions respectively. 'L' is the length of the transition, measured axially; 1/2 $B_{(t)}$ and 1/2 B are the half widths of contracted (throat) and full sections respectively.

5.4. The Author derived several other forms of transitions also based on assumptions regarding variation of velocity, water surface and boundary conditions. Details of these assumptions along with the resultant curves are presented in Author's Master's thesis(11).

6. DESIGN OF CONTRACTING TRANSITION

6.1. As already discussed, the sub-critical flow through a contracting transition is always stable, pressure gradient being favourable. Unless the velocity is very high or the constriction is too severe, the Author is of the opinion that any suitable streamlined shape of transition will suffice. The length of the transition required will depend on the degree of constriction adopted, the approach flow condition, the type of structure and the amount of head loss permissible. For low velocity unimportant structures, an average side splay of 1 : 1 to 2 : 1 will suffice. For important high velocity fumes, a length governed by an average side splay of 3 : 1 to 4 : 1 will be all right. Using too long a length or too complicated a shape of inlet transition wlill not only be costly and difficult to construct, but may be inefficient too, from the viewpoint of head loss, surface condition and velocity distribution, due to which the performance of the expanding transition also may be highly unsatisfactory. From the model studies made by Gibson(4), Formica (3) and others it has been found that if properly streamlined, the performance of contracting transition cannot be improved by adding more length beyond a certain limit governed by the flow characteristics and boundary conditions.

6.2 As regards the shape of contracting transition, it is recommended that it should be tangential to the wall at throat where the velocity is high. And for this matter use may be made of an elliptic quadrant, or any cylindrical surface with centre lying on the throat section. Use may also be made of *trocoidal* curves(9), where both inlet and outlet ends of the transition are desired to be made tangential to side walls.

6.3 In the absence of rigorous three-dimensional solution of motion in a contracting transition, the Author prescribes the following simple design procedure based on one-dimensional analysis :

(*i*) Fix up the length $L_{(trans)}$ of transition, depending on the allowable head loss, amount of constriction adopted and the maximum velocity in the constriction.

(*ii*) From the known ratio of constriction to be decided by the approach flow condition and economy; constricting beyond

$$F_{r_{(t)}} = (\overline{v_{(t)}} / \sqrt{g y_{(t)}} = 0.7$$

is often found to be uneconomic. Moreover for values of $F_{r(t)}$ 70.7, the flow is brought to the verge of critical stage when the surface becomes wavy, the widths at the beginning (uncontracted width$=B_1$) end and (throat width$=B_{(t)}$ of the transition are known.

(*iii*) Compute the depths, y_1 and $y_{(t)}$ and mean velocities, v_1 and $v_{(t)}$ at the beginning and end of transition respectively, from continuity and specific energy principles. (As first trial assume head loss to be nil and finally assume a suitable value of $C_{J(inl)}$— depending on the type of transition and flow conditions).

(*iv*) Plot the widths B and $B_{(t)}$ and the corresponding depths of flow y_1 and $Y_{(t)}$ at the respective sections, separated by the distance, $L_{(trans)}$ (fixed end points).

(*v*) Divide the transition length into a number of equal steps (depending on amount of length) and assuming that the mean velocity vary linearly from $\overline{v}_1$ to $\overline{v}_{(t)}$ over the length, $L_{(trans)}$ find the mean velocity at all the steps. Obviously the mean velocity ($\overline{v}_x$) at any section at distance '*x*' away from the entry will be :

$$\overline{v_x} = \overline{v_1} + (\overline{v_{(t)}} - \overline{v}_1) \times X/L_{(traxs)}$$

(*vi*) Compute the cross-sectional area ($A_\times$) required at all the steps for the given discharge, Q,

$$A_\times = \left(\frac{Q}{\overline{v}_\times}\right)$$

(*vii*) Since, $A_x = B_x \times y_x$, compute different combinations of B_{x1}', y_{x_1}; B_{x_2}, y_{x_2} and so on at all the steps corresponding to the respective areas of cross sections required (from step *vi*), keeping in mind the end values B_1, y_1 and $B(_t)$, $y(_t)$ (the intermediate widths, B_x, will vary between B and $B_{(t)}$ and the depths, y_x, between y_1 and $y_{(t)}$.

(*viii*) Plot the probable combinations of widths and corresponding depths at all the steps either using digits 1.1; 2.2; 3.3; or using same symbols (*e.g.*, 0, △, ▭, etc.) for the corresponding points in plan and elevation, so that looking at any section one may easily get the depth corresponding to any width chosen (Figure 1).

(*ix*) Join the corresponding points in plan and elevation, by trial and error, with the fixed end points, so that the transition (in plan) and the corresponding water surface profile (in elevation) are both smooth and continuous. The procedure of design for the contracting transition is illustrated by an example given in the appendix.

7. DESIGN OF EXPANDING TRANSITION

7.1. As already stated in para 3, sub-critical flow through the expanding transition is inherently unstable, pressure gradient being adverse. Although a number of curves had been evolved by several workers in the past, based on one-dimensional analysis, their performances have been far from satisfactory. Almost in every case, the flow separates from the boundary, resulting in head losses through production of turbulence. Another bad effect of separation is that the velocity distribution after exit of the expansion becomes extremely non-uniform, as the result of which materials at bed and sides of the channel, if unlined, get scoured away. In an attempt to determine the optimum length(10), the Author studied different lengths of transition governed by side splays varying from 0 : 1 to 10 : 1. It was found that the optimum splay for minimum loss in head varies between 7 : 1 to 9 : 1, depending on the initial flow condition (at throat) and the boundary conditions. Even at the optimum length, the head loss was pretty high, the velocity distribution at the exit was poor and the emergent flow was unstable and separating.

7.2. Use of too long a length with complicated shape of expanding transition should be avoided, since it is costly and its performance also is not satisfactory. The problem of flow expansion should be looked from the

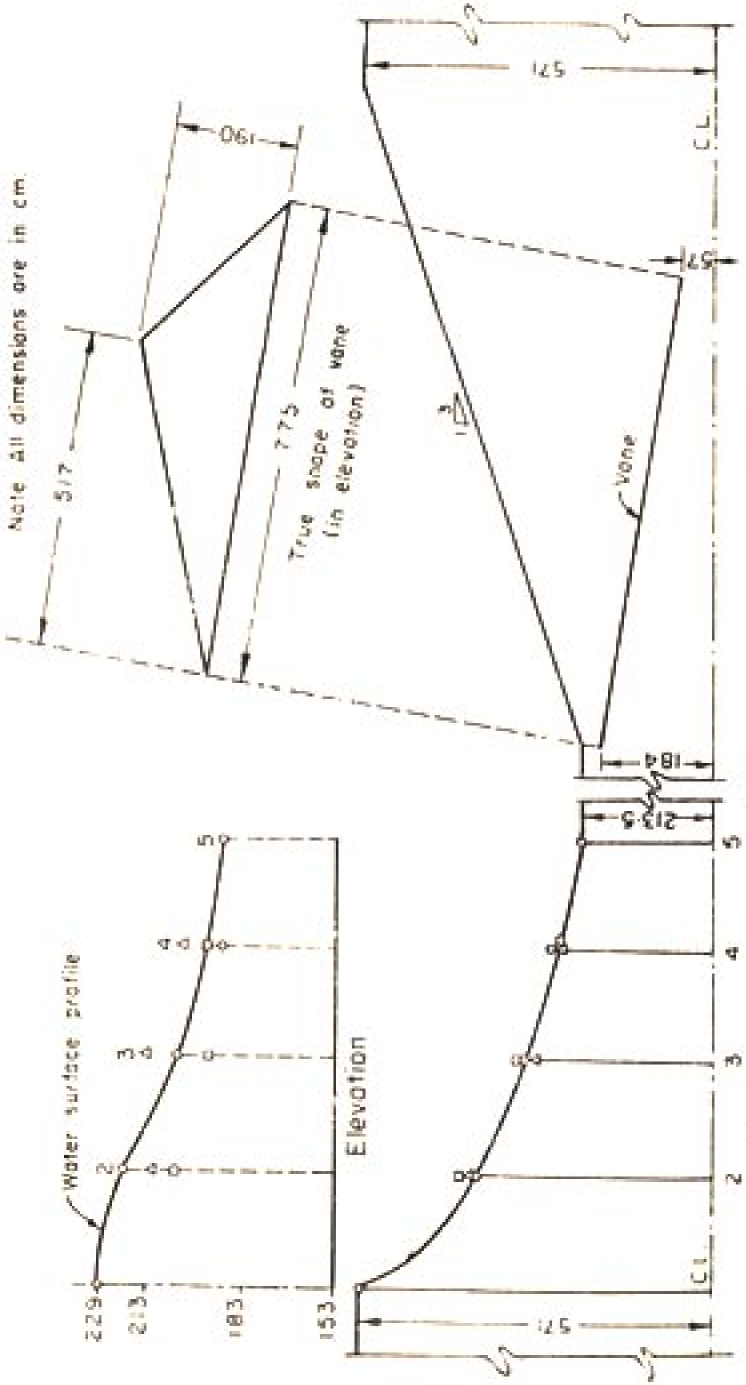

FIGURE 1 : Design of contracting and expanding transition

viewpoint of boundary layer flow control. As in aeronautics, suitable device should be adopted so that the boundary layer separation may be prevented or delayed. Amongst several workers who have contributed towards this aspect of open channel expansion design, mention may be made here of Smith and Yu(¹¹), Kline and Cochran(⁸), Rao(¹¹) and others(¹²) (¹³) (⁶). The Author in his Ph. D. thesis, took up the problem of "Design of wide-angle open channel expansions in sub-critical flow by control of boundary layer separation with triangular vanes". In this new method, a pair of submerged triangular vanes were placed symmetrically and converging downstream in plan, at the commencement of expansion (Figure 1), in order to divert the high velocity central streamlines to the sides to avoid separation and eddying. After testing different orientations and geometry of the vanes in about 600 experimental runs, the optimum geometry of the vanes, their spacing and inclination were developed for a given expansion [from 2.7 to 7.3 cm (1.06 to 2.87 inch) at the rate of 3 : 1 straight flaring] with given entrance condition. Nine different flows (three discharge and three different depths for each discharge) were tested and the optimum geometry was developed for all the flows. Efficiency of the vanes expansion was defined by three independent criteria, namely,

(*i*) The hydraulic efficiency, $\eta_{(H)}$, defining the loss in head :

$$\eta_{(H)} = \frac{(y_2 - y_{(t)})}{\left(\frac{v_{(t)}^2}{2g} - \frac{v_2^2}{2g}\right)} \times 100 \; ; \; (C_{f_{(om)}} = 1 - \eta_{(H)}/100$$

(*ii*) The standard deviation (x_{sd}) of the exit bed shear distribution with respect to the normal bed shear distribution corresponding to uniform and parallel flow. The parameter "x_{sd}" gives a measure of the non-uniformity of velocity distribution and scour in the tail channel after expansion.

(*iii*) The separation and stalling pattern indicating the disturbance and smoothness of the exit flow.

7.3. From the graphical plots of the hydraulic performances in respect of hydraulic efficiency, exit bed shear distribution and stall characteristics, optimum values of the height, length, upstream spacing and inclination of the vanes, all expressed non-dimensionally, were developed for various discharges and depths of flow (covering the practical range). Design curves for the optimum parameters, of the vanes are given in Figures 2 (*a*) and 2 (*b*) for maximum hydraulic efficiency. It is not possible to incorporate here all the performance curves, for the various geometry and for the different flows tested due to want of space. However, the results achieved by chosing the optimum dimension of the vanes have been compared in Figure 3 with those in other standard designs.

7.4. The hydraulic efficiency which was extremely small (23.6 per cent on an average) for straight plain expansion (3 : 1 side splay) was increased to a great extent (71 per cent on an average) by providing the vanes and was greater than the maximum efficiency (66 per cent on an average) obtained in plain curved expansion at optimum length (governed by average side

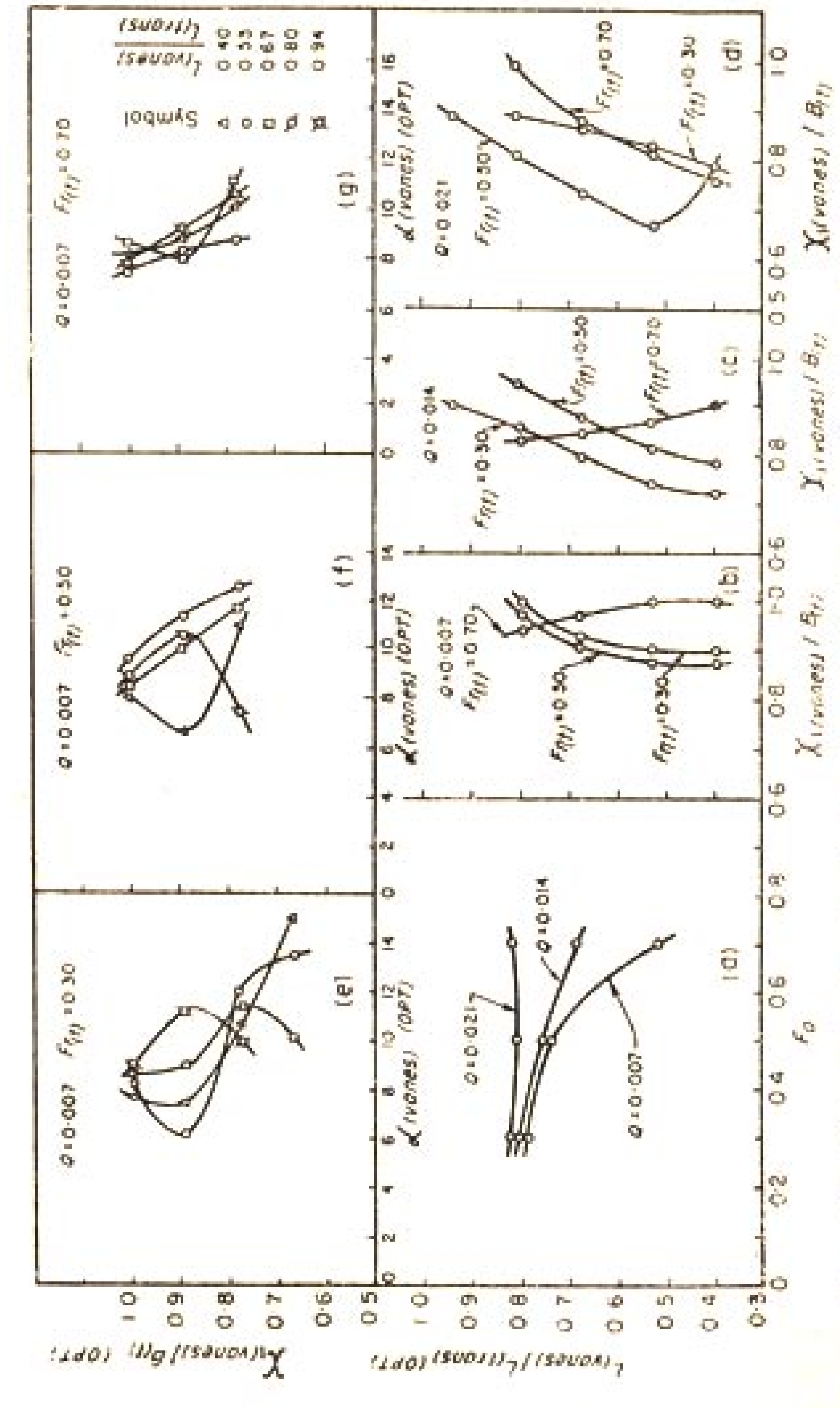

FIGURE 2(*a*): Design curves of optimum length, spacing and inclination of vanes for maximum hydraulic efficiency.

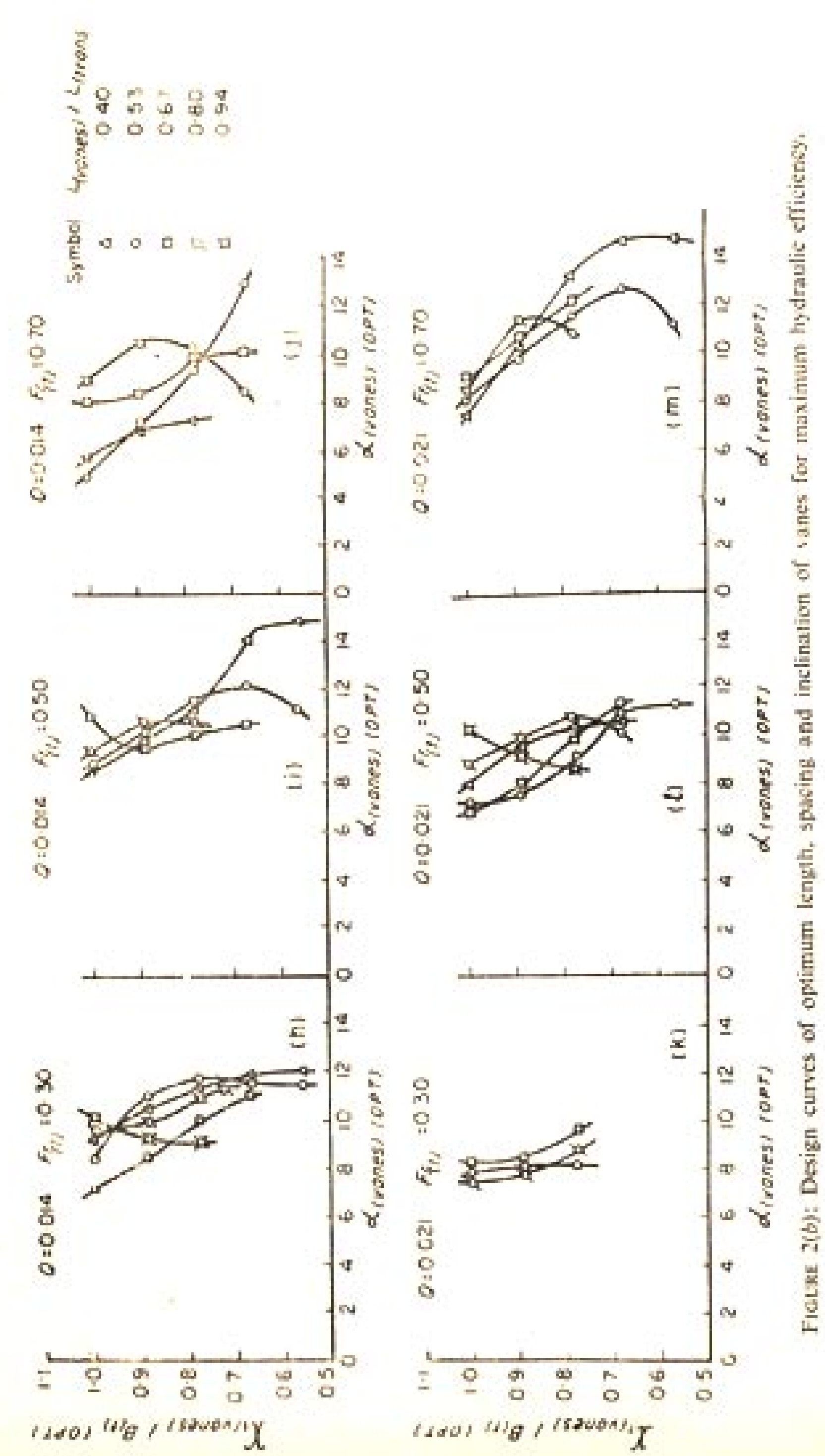

FIGURE 2(*b*): Design curves of optimum length, spacing and inclination of vanes for maximum hydraulic efficiency.

splay lying between 7 : 1 and 9 : 1). Velocity and exit bed-shear distribution which was highly distorted (x_{std}=2.5 on an average) and caused large amount of scour in the tail channel in plain expansion (3 : 1 side splay) was improved remarkably (x_{std}=0.25 on an average) and made almost the same as in normal uniform flow, upon insertion of the vanes, with the result that the scour was completely eliminated. Most promising results were obtained in respect of separation and stalling of flow. Without vanes, the flow was found to separate right at entry and stall violently within the expansion and the issuing jet was found to swing from one side to the other (the flow being extremely unstable), the opposite side being occupied by huge body of stall. On insertion of the vanes, however, the separation was completely eliminated and the stall disappeared totally, resulting in smooth and very stable flow in the tail channel. Figures 4 and 5 depict the scour pattern after the expansion with and without vanes.

7.5 Procedure for design of an expanding transition has been illustrated by an example given in Appendix I.

8. CONCLUSIONS

(*i*) Providing too long a transition with too much complicated shape is neither economical nor does it secure greater efficiency.

(*ii*) Sub-critical flow in a contracting transition being stable, any suitable streamlined shape will suffice for it.

(*iii*) A new method, based on linear change in mean velocity, is recommended for the design of contracting transition.

(*iv*) Flow in an expanding transition is unstable. Separation occurs at sides leading to high head loss and non-uniform distribution of velocity due to which there occurs large amount of scour in the tail channel.

(*v*) The problem of flow expansion should be looked from the view-point of boundary layer control. Attempt should be made to prevent or delay boundary layer separation by providing suitable appurtenances.

(*vi*) A new method of design for expansion using a pair of short submerged triangular vanes placed symmetrically and converging downstream in plan at the commencement of expansion is recommended. The design is simple (straight walls), economic (only 3 : 1 side splay) and highly efficient (low head loss, free from scour and separation).

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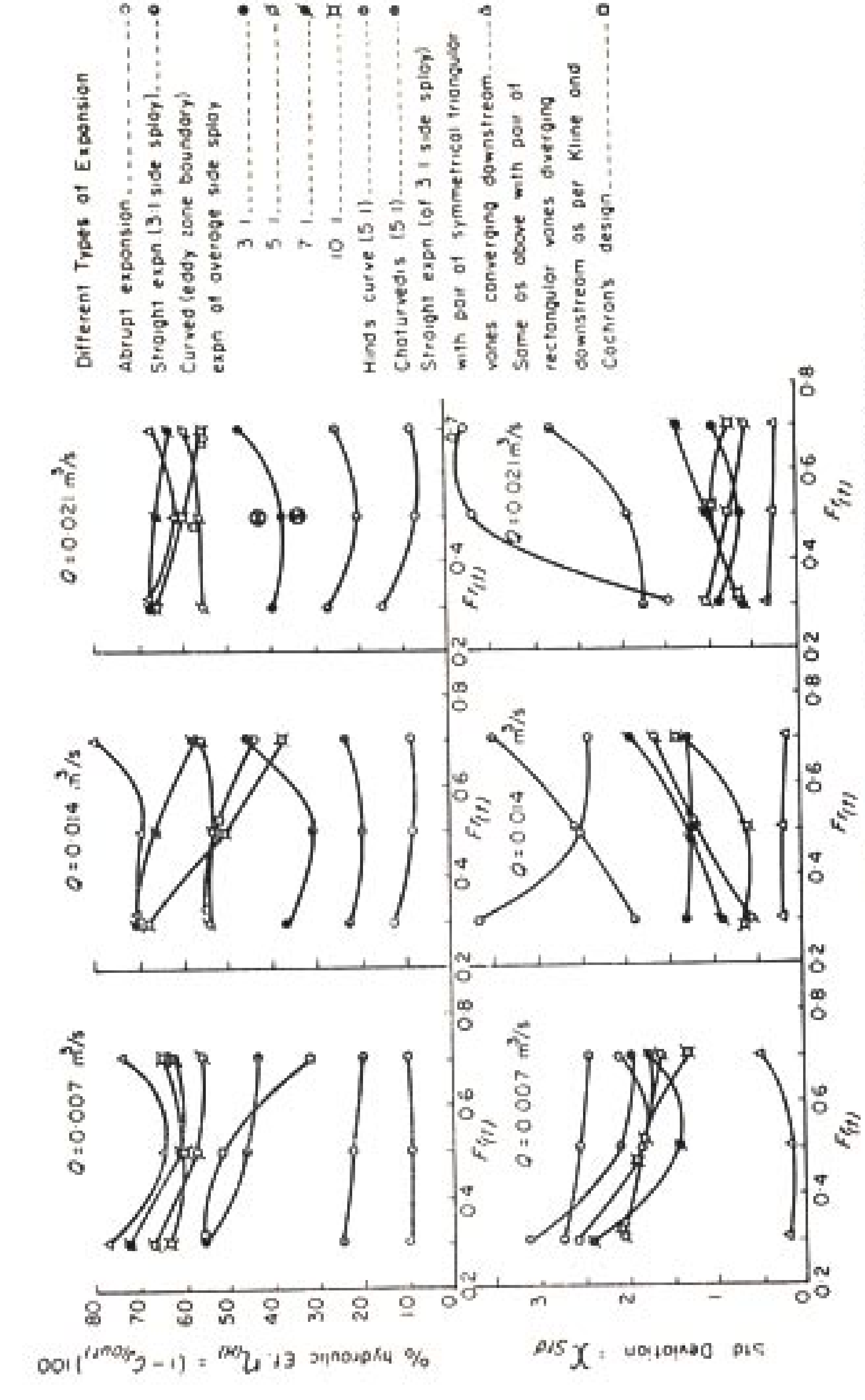

FIGURE 3: Comparison of hydraulic efficiency and standard deviation for different types of expansions

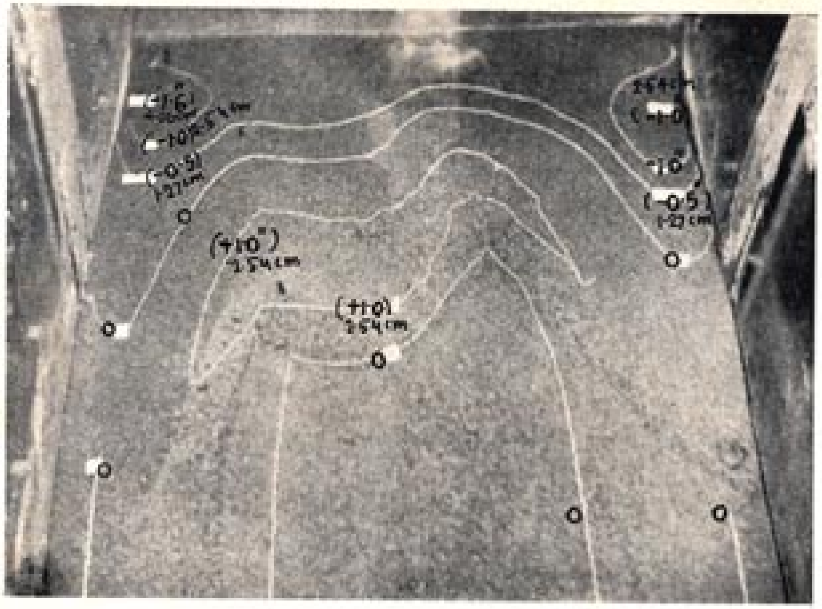

FIGURE 4: Showing scour pattern with vanes ($Q=0{\cdot}02$ m³/s, $F_{r(+)}=0{\cdot}50$) time = 30 minutes)

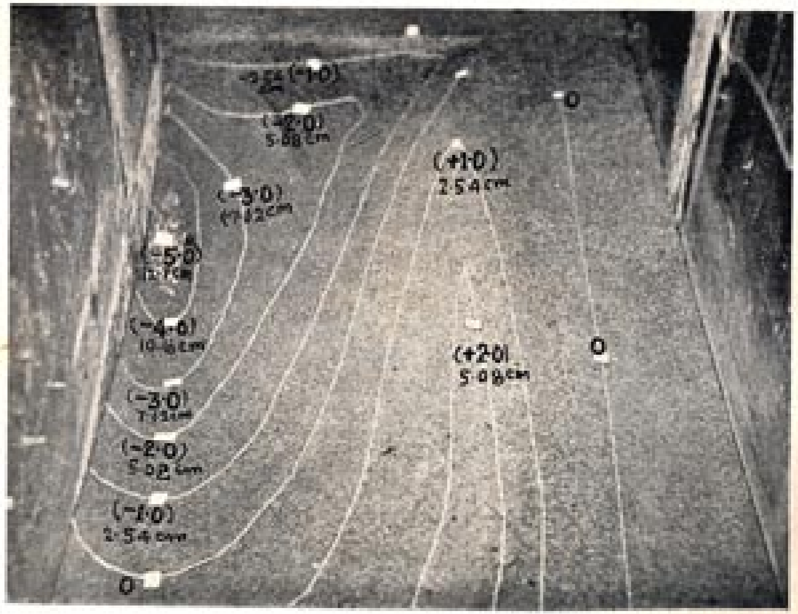

FIGURE 5: Showing scour pattern without vanes ($Q=0{\cdot}02$ m³/s, $F_{r(+)}=0{\cdot}50$, time = 30 minutes)

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APPENDIX I

DESIGN EXAMPLE

Design the contracting and expanding transition for an aqueduct in an irrigation canal, with data given below :

(*i*) Full supply discharge, $Q=23.5$ m³/s (830 causec)
(*ii*) Uncontracted width, $B_1=B_2=11.42$ m (37.5 feet)
(*iii*) Full supply depth, $y_1=y_2=2.29$ m (7.5 feet)
(*iv*) Normal velocity of flow, $v_1=v_2=0.90$ m/s (2.95 ft/s)

Maximum economic constriction ratio = 2.67 (Determined from graphical plot of constriction ratio, $B/B_{(t)}$, against Froude's Number at throat, $F_{r(t)}$.

∴Width at throat, $B_{(t)}=11.42/2.67=4.27$ m (14 feet). Assuming head loss in contracting transition to be nil,

$$E_{(t)}=E_1=2.29+\left(\frac{23.5}{11.42\times2.29}\right)^2\times\frac{1}{2\times9.81}=2.33\text{ m}\,(7.644\text{ ft})$$

$$\text{or} \quad y_{(t)} + v^2_{(t)}/2g = y_{(t)} + \left(\frac{23.5}{4.27 \times y_{(t)}}\right)^2 \times \frac{1}{2 \times 9.81} = 2.33$$

Solving the cubical equation in $y_{(t)}$ above, by trial,

$y_{(t)} = 1.90$ m/s (6.22 ft)

$v_{(t)} = 2.90$ m/s (9.51 ft/s)

$F_{r(t)} = v_{(t)}/\sqrt{gy_{(t)}} = 0.68 < 0.70$, o.k.

DESIGN OF CONTRACTING TRANSITION

Assuming a length of transition governed by average 2 : 1 side splay, axial length of contracting transition $= \frac{2\,(11.42 - 4.27)}{2} = 7.15$ m (23.5 feet). Divide the total length of transition into 4 equal steps and number the sections as 1—1, 2—2, 3—3, 4—4 and 5—5 (Figure 1). Total change in velocity = 2.90—0.90 = 2.0 (6.57 ft/s).

∴ Change in velocity between two consecutive sections = 2.0/4 = 0.5 m/s (1.64 ft/s).

TABLE

Probable widths of transition and corresponding depths of flow

Section No.	*Mean velocity m/s ft/s*	*Area of section m² (ft²)*	*Probable widths m (ft)*	*Corresponding depths m (ft)*
1—1	0.90 (2.95)	26.2 (282)	11.42 (37.5)*	2.29 (7.5)*
2—2	1.40 (4.59)	16.8 (181)	7.63 (25.0) 7.93 (26.0) 8.23 (27.0)	2.21 (7.25) 2.12 (6.96) 2.04 (6.69)
3—3	1.90 (6.23)	12.35 (133)	6.40 (21.0) 6.10 (20.0) 5.79 (19.0)	1.934 (6.34) 2.03 (6.66) 2.135 (7.0)
4—4	2.40 (7.87)	9.81 (105.6)	5.18 (17.0) 5.03 (16.5) 4.87 (16.0)	1.892 (6.2) 1.95 (6.40) 2.016 (6.60)
5—5	2.90 (9.31)	8.10 (85.2)	4.27 (14.0)*	1.90 (6.23)*

* Fixed end points.

Plot the end fixed points on sections 1—1 and 5—5 both in plan and elevation, as shown in Figure 1.

From the above table plot the probable widths and corresponding depths of flow by using same symbol in plan and elevation as shown in Figure 1 at all intermediate sections 2—2, 3—3 and 4—4.

Join the corresponding points in plan and elevation with the end fixed points by trial and error, so that the transition (in plan) and the corresponding water surface profile (in elevation) are both smooth and continuous (Figure 1).

DESIGN OF EXPANDING TRANSITION

Width at throat of prototype = 4.27 m (14.0 ft).

,, ,, of model (expt.) = 0.229 m (9 in.)

$$\therefore \text{ Model ratio} = r_{(mod)} = \frac{4.27}{0.229} = 18.65$$

$$Q_{(prot)}/Q_{mod} = r_{(mod)}^{\frac{5}{2}} = (18.65)^{5/2} = 1500$$

$$\therefore \quad Q_{(mod)} = 23.5/1500 = 0.0156 \text{ m}^3\text{/s } (0.55 \text{ ft}^3\text{/s})$$

$F_{r(t)}$ = Froude's Number at throat = 0.68

From design curves [Figure 2(*a*), and 2 (*b*)] for maximum hydraulic efficiency, optimum dimensions of the vanes corresponding to $Q = 0.0156$ and $Fr_{(t)} = 0.68$, are :

$L_{(vanes)}\ L_{(trans)} = 0.72$, $U/B_t = 0.86$, $\alpha_{(vanes)} = 9.4°$ and $H/Y_t = 1.0$

∴ $L_{(trans)}$ = Length (axial of expanding transition (with 3:1 side splay)

$$= \frac{3(11.42 - 4.27)}{2} = 10.73 \text{ m } (35.25 \text{ ft})$$

∴ $L_{(vanes)}$ = total length of vanes at base = $0.72 \times 10.75 = 7.75$ m (25.5 ft) with 2 : 1 base ratio, length at apex of vane = 5.17 m (17 ft), $Z_{(vanes)}$ = Upstream spacing of vanes (out to out) at entry = $0.86 \times 4.27 =$ 4.68 m (12 ft)

$\alpha = {}_{(vanes)}$ inclination of the vane with the axis = 9.4°,

∴ $\alpha_{(vanes)} \sin \alpha_{(vanes)} = 1.27$ m (4.17 ft)

∴ Distance between centre line and the outer face of vane at the toe

$$= Z_{1(vanes)/2} - L_{(vanes)} \sin \alpha) = \left(\frac{3.68}{2} - 1.27\right)$$

$= (1.84 - 1.27) = 0.57$ m (1.85 ft).

$H_{(vanes)}$ = height of the vane at apex = $y_{(t)} = 1.90$ m (6.23 ft)

Thickness of vane = $\gamma_{(mod)} \times 6.4$ mm = 120 mm (4.72 inches).

Detailed dimensions of the vanes with orientation and spacing are shown in Figure 1.

Head loss (with vanes) : $\eta_{(H)} = 77.5$ per cent (from Figure 3 for $C = 0.0156$ and $F_{r_{(t)}} = 0.68$)

$$\therefore \quad C_{J_{(out)}} = 1 - 0.775 = 0.225,$$

$$\therefore \quad J_{(vanes)} = 0.225 \times \frac{1}{2 \times 9.81} \left[(2.90)^2 - (0.90)^2\right] = 0.087 \text{ m } (0.280 \text{ ft})$$

Head loss (without vanes): $\eta_{(H)} = 22.7$ per cent (from Figure 3) for $Q = 0.0156$ and $F_{r_{(t)}} = 0.68$)

$$\therefore \quad C_{J_{(out)}} = 1 - 0.227 = 0.773.$$

$$\therefore \quad J_{(vanes)} = 0.773 \times \frac{1}{2 \times 9.81} \left[(2.90)^2 -- (9.90)^2\right] = 0.30 \text{ m } (0.984 \text{ ft})$$