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LIMIT OF SUBMERGENCE IN CRITICAL FLOWMETERS*

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Summary

This paper analyzes certain aspects of flow in a critical flowmeter and determines the 'modular limit' or 'limit of submergence' for the meter for various constriction ratios and exit conditions. The limit of submergence is proved to be governed by the head losses at the entry and exit and due to friction. It also attempts to correlate these losses with the submergence limit and derives a mathematical expression for the same. A set of curves illustrating the point of submergence for various constriction ratios, and entry and exit conditions are proposed and the same verified by experiments. A new type of transition, used for the entry and the exit, was found to be efficient.

Notations

B = unconstricted width of channel,

b = constricted width of channel,

$r = \frac{b}{B}$ = constriction ratio,

Y_1 = upstream depth of water, measured at a point where the effect of draw-down is nil,

Y_2 = downstream depth of water, measured at a point where the water surface is brought back to normal condition after formation of standing wave,

$Y_c = \left[\frac{\left(\frac{Q}{b}\right)^2}{g}\right]^{\frac{1}{3}}$ = critical depth of water,

Q = total discharge flowing through the channel,

q = intensity of discharge per ft. width,

$\mu = 2 \times \sqrt{\frac{1}{r^3}\cos^3\left(\frac{\pi}{3} + \frac{1}{3}\cos^{-1} r\right)}$ = constant for a given meter,

C = coefficient of discharge in the critical flowmeter,

$S_{cr.} = \frac{\lambda_1}{\lambda_2}$ = critical submergence ratio,

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$\lambda_1 = \frac{Y_c}{Y_1}$,

$\lambda_2 = \frac{Y_c}{Y_2}$,

E_{f1} = upstream specific energy of flow,

E_{f2} = downstream specific energy of flow,

$b_{cr.}$ = width of constriction corresponding to the critical flow under normal state of flow without any heading up,

q_{bm} = maximum intensity of discharge that may be allowed for critical flow under the normal state, *i.e.*, without any heading up on the upstream side,

$q_b = \frac{Q}{b}$ = intensity of discharge per ft. width of actual constriction width b,

$q_B = \frac{Q}{B}$ = intensity of discharge per ft. width of unconstricted channel,

$Y_{cr.}$ = critical depth of water corresponding to $b_{cr.}$,

H_L = total head loss in the meter, *i.e.*, summation of losses in entry, exit, friction and in standing wave,

g = uniform acceleration due to gravity,

$H_{Li} = C_i\left(\frac{V_c^2}{2g} - \frac{V_1^2}{2g}\right)$ = head loss at entry,

$H_{Lo} = C_0\left(\frac{V_c^2}{2g} - \frac{V_2^2}{2g}\right)$ = head loss at exit,

C_i = coefficient of head loss at entry,

C_0 = coefficient of head loss at exit,

$C_1 = (1 + C_i)$,

$C_2 = (1 - C_0)$, and

K = a constant for actual critical submergence ratio.

1. Introduction

Critical flowmeter is the latest device for measuring discharges in open channels. Unlike weirs and notches, here the constriction is made horizontally, so that critical state of flow occurs in the constricted portion. Under this condition, the discharge may be expressed as function of upstream depth of water. The discharge formula[1] is given by

$$Q = C_d\, b\, Y_1^{\frac{3}{2}} \tag{1}$$

where

$$C_d = C\,\mu\sqrt{2g}$$

$$\mu = 2\sqrt{\frac{1}{r^3}\cos^3\left(\frac{\pi}{3} + \frac{1}{3}\cos^{-1} r\right)}$$

and r is the constriction ratio, b the width of constriction, C the coefficient varying with constriction ratio, entry condition, roughness, etc., and Y_1 the upstream depth of water, measured at a place where there is no drawdown due to entry.

An important aspect of critical flowmeter is its 'modular limit' or 'limit of submergence', which may be defined as the highest ratio of downstream depth of water to the upstream one without affecting the state of flow on the upstream side. The free flow formula given by equation (1) will hold good only up to the limiting submergence. Thus, 'submergence limit' draws a line of demarcation between free and submerged flow.

2. Historical note

Most authors dealing with metering of flow in an open channel have recognized the effect of submergence. Parshall[2], while suggesting his formula, stated : 'For the most satisfactory operation of this device, it is recommended that the flume operate under conditions in which the submergence does not exceed 70%'.

In the discussion of Parshall's paper, Hinds[3] has presented a table showing the percentage of submergence at the instant of flooding for his typical structures. The average value of his investigation was found to be 77%, although individual variation ranged between 60-89%.

Addision[4] added that : 'The downstream flared part of the flume has no effect on the head discharge relationship, but it materially helps to secure a high ratio of submergence or modular limit. Although this may sometimes be as much as 0.92, it is prudent not to rely on a value higher than 0.7'.

Villemonte[5] in comparing the equation for submerged weirs with a 6-in. Parshall flume, has given an experimental curve, which shows that up to a submergence of 0.56 the water surface is not affected by any increase in tail water. This point was termed as 'effective submergence'. 'Critical submergence' in his experiment occurred at about 90% submergence when a series of standing waves formed in the diffused zone.

It is apparent from this discussion that though the effect of submergence was recognized by the previous authors, no attempt was made to correlate the same with meter characteristics.

3. Balloffet's formula

Balloffet[2] developed an expression for the critical submergence given by

$$S_{cr.} = \frac{Y_1}{Y_2} = \sqrt[3]{2\,\mu^2}\left(\frac{3}{2} - 0.33787\, r^{\frac{2}{3}}\right) \tag{2}$$

where Y_2 is the downstream depth of water and $S_{cr.}$ the critical submergence ratio.

Equation (2) reveals that the 'critical submergence' varies only with the constriction ratio r, of the meter. But it is found from experiment that it varies widely with variation in entry and exit conditions also. Thus, the above equation cannot be applied as a general one, though it may be found valid for the typical cases analyzed by Balloffet.

Moreover, certain basic assumptions made by Balloffet may be far from the actual state of affairs. For example, the width of the jump at the point of submergence need not be equal to the unconstricted width of the channel. The relationship given by

$$Y_2 = -\frac{Y_w}{2} + \sqrt{\frac{Y_w^2}{4} + \frac{2\,Y_c^3}{Y_w}}$$

in standing wave, does not hold good at the point of critical submergence when the jump has been considerably submerged. The losses in entry and exit cannot be neglected. Lastly, the difference between the upstream energy level and the tail water is not absolutely the minimum at the point of critical submergence, as assumed in the derivation.

4. Flow through critical flowmeter

Before taking up the problem of 'critical submergence', it will be useful to discuss certain aspects of flow through the critical flowmeter connected with the submergence.

Referring to Fig. 1, let Y_2 and E_{f2} be the normal depth and specific energy of flow in a channel, before any constriction is adopted. The discharge per ft. width of the channel q_B, is given by $\frac{Q}{B}$, where Q is the total discharge and B the unconstricted width of channel. In Fig. 2, point P_B on E_{f2} curve represents this flow which is subcritical for the given conditions.

The normal state of flow, without any heading up, will exist in this channel, so long as E_{f2} remains the same. Therefore, the maximum intensity of discharge that can be allowed through the channel without heading up is given by q_{bm} (Fig. 2). In other words, the maximum amount of constriction that may be adopted in the channel for the given discharge Q, and no heading up is given by $b_{cr.}$ which equals to $\frac{Q}{q_{bm}}$. This critical width has been indicated by dotted lines in Fig. 1, and the corresponding water profile with $Y_{cr.}$ as the critical depth has also been shown in Fig. 1.

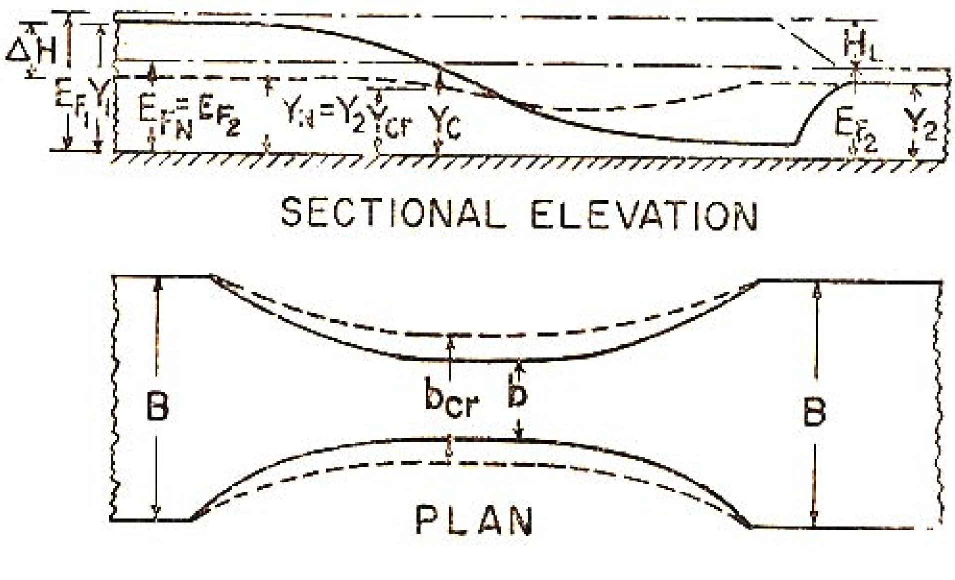


Fig. 1

Specific discharge diagram

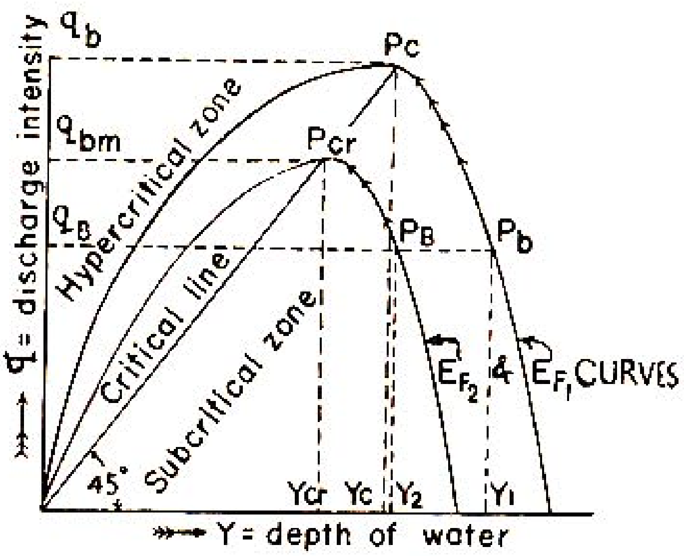


Fig. 2

Curves of discharge intensity *vs.* depth of water

The actual width of constriction b of a 'critical flowmeter' is less than b_{cr}, because the meter has to be used for all stages of flow. So q_b, the actual intensity of discharge, is more than q_{bm}. To allow q_b, the specific energy of flow should rise to some optimum value E_{f1}, such that a horizontal through q_b just touches the E_{f1} curve. The heading up necessary to achieve this increment of specific energy and allow critical flow through the constriction has been indicated in Fig. 1. Then Y_1 becomes the new upstream depth, corresponding to the discharge intensity q_B in the unconstricted channel. The corresponding critical depth is Y_c. Point P_b gradually shifts to point P_c on the specific energy curve as q changes from q_B to q_b. Obviously, the flow is subcritical in this upstream region.

Downstream of the constriction, where flow is normal, E_{f1} is brought back to E_{f2} through necessary head loss $(E_{f1} - E_{f2})$. If the losses in entry, exit and friction is not sufficient to destroy the extra hydraulic energy $(E_{f1} - E_{f2})$, standing wave shall form dissipating the balance. The flow, in that case, changes to supercritical state. Point P_c will shift somewhere on the supercritical limb of the E_{f1} curve, and will be brought back to P_B on the subcritical limb of E_{f2} curve, through the formation of hydraulic jump.

5. Head losses and limit of submergence

For a given constriction width b, the amount of specific energy upstream, E_{f1}, is fixed for a given discharge. Because,

$$E_{f1} = \frac{3}{2} Y_c = \frac{3}{2} \left(\frac{q_b^2}{g}\right)^{\frac{1}{3}} \tag{3}$$

or,

$$(E_{f2} + H_L) = E_{f1} = \text{Constant} \tag{4}$$

where H_L is the total head loss in between sections (1) and (2).

The head losses consist of :

(i) Loss in entry ;

(ii) Loss in exit ;

(iii) Loss in friction ; and

(iv) Loss in hydraulic jump.

As already stated, the 'critical submergence' of a flowmeter arises just when the upstream depth begins to change with any further increment in tail water. As the tail water is gradually raised, E_{f2} increases and H_L decreases [from equation (4)]. The submergence limit will, therefore, be determined by the extent to which the head losses can be reduced by increasing the tail water depth, but keeping $(E_{f2}+H_L)$ constant, as given by equation (3). After the point of submergence, if the tail water is further increased, the head loss is not correspondingly diminished and the total of E_{f2} and H_L exceeds E_{f1} as given by equation (3); heading up occurs, and the whole flow changes to subcritical stage. The meter in such a case will be submerged.

If the width of the meter is made exactly equal to $b_{cr.}$ and if the losses in entry, exit and friction can be reduced almost to zero by suitable constriction of the meter, H_L tends to zero, because there is no standing wave formation in this ideal width constriction. Thus, under this idealized case, the submergence limit tends to 100%. Because, as H_L tends to zero, E_{f2} almost equals E_{f1} and $\frac{Y_2}{Y_1}$ tends to unity.

If the width, however, is less than $b_{cr.}$ there will be heading up on the upstream side, with the formation of standing wave necessary to destroy 'the balance of energy' left after frictional and transitional losses. But as the tail water is gradually raised, the standing wave front moves forward, the jump gets more and more submerged, and finally at the point of critical submergence, jump is totally suppressed and there is no loss due to hydraulic jump because the 'balance of energy' which was left in normal state of flow has now been compensated by an equal increment in E_{f2} through the artificial tail water control.

The loss due to hydraulic jump being nil at the 'critical submergence' condition, losses in transition and friction alone govern the point of limiting submergence.

Thus, in any flowmeter, where transitional and frictional losses can be reduced almost to zero, the limit of submergence is almost 100%, as before.

In actual practice, however, there will be certain amount of losses in friction and transition even if the transition is most scientifically designed and the smoothest possible construction adopted. The head loss H_L, will, thus, have some finite value and E_{f2} will be less than E_{f1} for all possible flows. So, $\frac{Y_2}{Y_1}$ will be less than unity, *i.e.*, the submergence limit will be below 100%.

6. Mathematical expression for limit of submergence

Assumptions

(i) Frictional loss is neglected, being very small in comparison with transition losses.

(ii) Floor of the meter is assumed in level with that of the critical section.

(iii) H_{Li} = Loss at entry $= C_i^7 \left(\frac{V_c^2}{2g} - \frac{V_1^2}{2g}\right)$ (4)

(iv) H_{Lo} = Loss at exit $= C_o^8 \left(\frac{V_c^2}{2g} - \frac{V_2^2}{2g}\right)$ (5)

where V_1 and V_2 are the average velocities of flow at upstream and downstream ends of the meter respectively and V_c is the average velocity at the critical section, and C_i and C_o are the coefficient of inlet and exit losses respectively. The values of C_i and C_o are assumed to vary with the nature of transitions alone.

With the assumptions, stated above, and with reference to Fig. 3 applying Bernoulli's equation, between upstream (1 - 1) and critical sections (C - C) at the limiting submergence condition, we get

$$Y_1 + \frac{V_1^2}{2g} = Y_c + \frac{V_c^2}{2g} + H_{Li} \quad (6)$$

or, from equation (4),

$$Y_1 = Y_c + \left(\frac{V_c^2}{2g} - \frac{V_1^2}{2g}\right) + C_i \left(\frac{V_c^2}{2g} - \frac{V_1^2}{2g}\right)$$

$$= Y_c + (1 + C_i)\left(\frac{V_c^2}{2g} - \frac{V_1^2}{2g}\right) \quad (7)$$

From equation (7),

$$C_i = \frac{Y_1 - Y_c}{\frac{V_c^2}{2g} - \frac{V_1^2}{2g}} - 1 \quad (8)$$

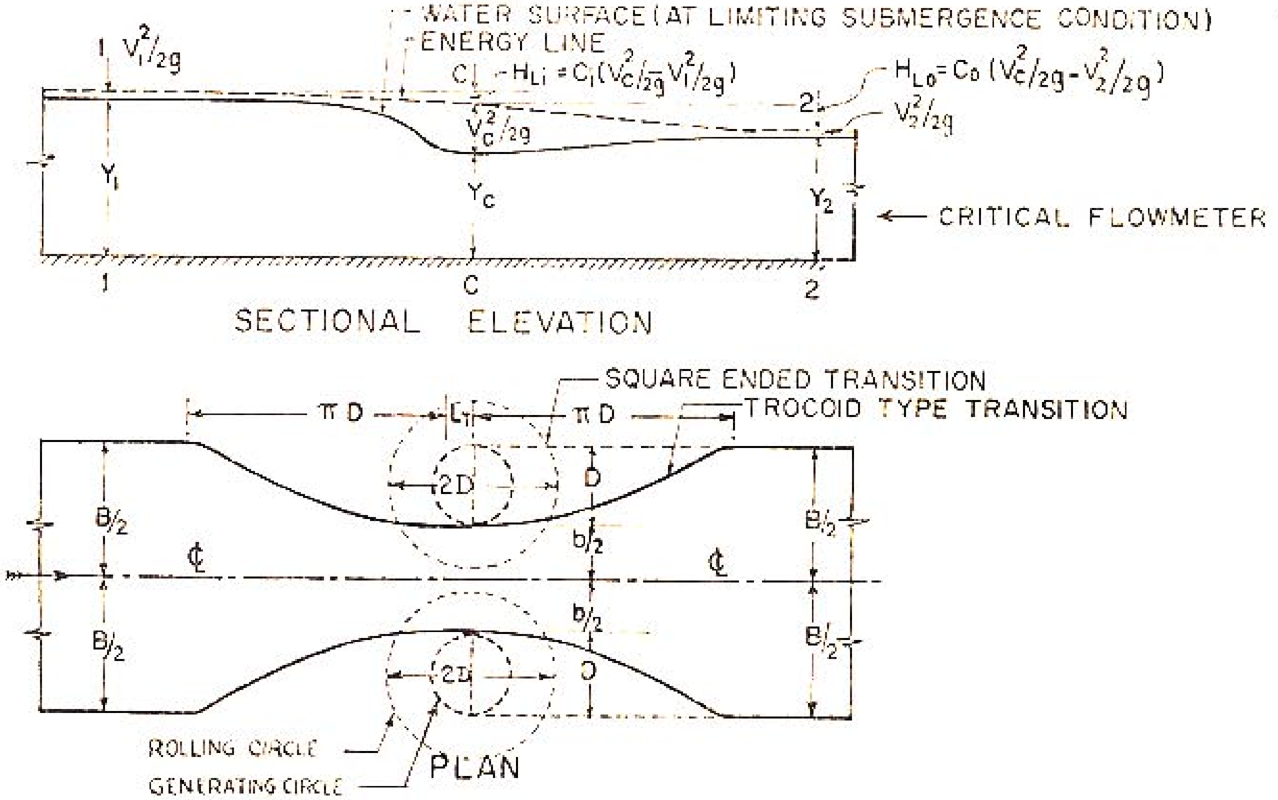


Fig. 3

Diagram for calculating the limit of submergence

Putting

$$V_1 = \frac{Y_c}{Y_1} r\, V_c \tag{9}$$

from equation of continuity and

$$\frac{V_c^2}{2g} = \frac{Y_c}{2} \tag{10}$$

from characteristic of critical flow, equation (7) can be written as

$$Y_1 = Y_c + (1 + C_i)\left(1 - r^2 \frac{Y_c^2}{Y_1^2}\right) \frac{Y_c}{2} \tag{11}$$

Dividing both sides by Y_1 and putting $\frac{Y_c}{Y_1} = \lambda_1$ and $(1 + C_i) = C_1$, equation (11) reduces to

$$\lambda_1^3 r^2 C_1 - \lambda_1 (2 + C_1) + 2 = 0 \tag{12}$$

Values of C_i and C_o for various types of transitions have been calculated by the United States Bureau of Reclamations for subcritical flow, which is obviously the case under critical submergence condition. Values of C_i and C_o have however been found to differ with constriction ratio and the percentage of submergence. The values under critical submergence condition have been experimentally determined and given in Table 3 for the models tested.

Similarly, applying Bernoulli's equation between critical section (C-C) and the downstream (2-2) at the limiting submergence condition, it can be proved that

$$\lambda_2^3 r^2 C_2 - \lambda_2 (2 + C_2) + 2 = 0 \tag{13}$$

where $\lambda_2 = \left(\frac{Y_c}{Y_2}\right)$, r is the constriction ratio, $C_2 = (1 - C_0)$ and

$$C_0 = 1 - \frac{Y_2 - Y_c}{\frac{V_c^2}{2g} - \frac{V_2^2}{2g}} \tag{14}$$

Therefore,

$$S_{cr.} = \frac{Y_2}{Y_1} = \frac{\frac{Y_c}{Y_1}}{\frac{Y_c}{Y_2}} = \frac{\lambda_1}{\lambda_2} \tag{15}$$

Taking the effect of frictional losses, etc., the exact value of 'critical submergence' will be given by

$$S_{cr.} = K \frac{\lambda_1}{\lambda_2} \tag{16}$$

where K is a constant, depending on the roughness and other characteristics of the meter such as form, throat length, etc.; the value of K is obviously less than unity.

The effect of curvature at entry and exit and the non-uniform velocity distribution may also be included in the constant K, because the analysis becomes extremely complicated when these dynamic effects are also included, and the precision so obtained is practically immaterial so far as the practical use of the meter is concerned.

7. Values of submergence limit for various types of transitions

The values of λ_1 and λ_2 may be found out by solving for the roots of the cubical equations in λ_1 and λ_2 given by equations (12) and (13) respectively, for various types of transitions adopted at the entry and exit of the meter, as C_1 and C_2 changes with change in C_i and C_o. The values of C_i and C_o, as given by the United States Bureau of Reclamations for the design of certain standard transitions, are given in Table 1.

Table 1

Values of C_i and C_o

Type of transition	C_i	C_o
Warped type	0.10	0.20
Cylinder quadrant	0.15	0.25
Wedge type	0.20	0.30
Straight line type	0.30	0.50
Square ended type	0.30	0.75

Since it is rather laborious and time consuming to solve the values of λ_1 and λ_2 for every individual case, the same may be readily obtained from graphical construction. Such graphs for the standard values of C_i and C_o have been plotted and presented in Figs. 4 to 7. Knowing the constriction ratio r, and the types of transition adopted at entry and exit, value of 'critical submergence ratio' can be read directly from these curves. For any intermediate value, the results may be interpolated without appreciable error.

It is interesting to note that when both C_i and C_o are zero, equations (12) and (13) are the same, *i.e.*, $\lambda_1 = \lambda_2$ or, the submergence limit is 100% for all values of r, the constriction ratio.

8. Experimental results

Experiments were carried out in the Hydraulics Laboratory of the Indian Institute of Technology, Kharagpur, for verification of the theoretical values of 'submergence limit'. Studies were made for :

(i) Constriction ratios $r = 0.4$, $r = 0.6$, $r = 0.8$;

(ii) Gradual and square ended types of expansion for each of the values in item (i); and

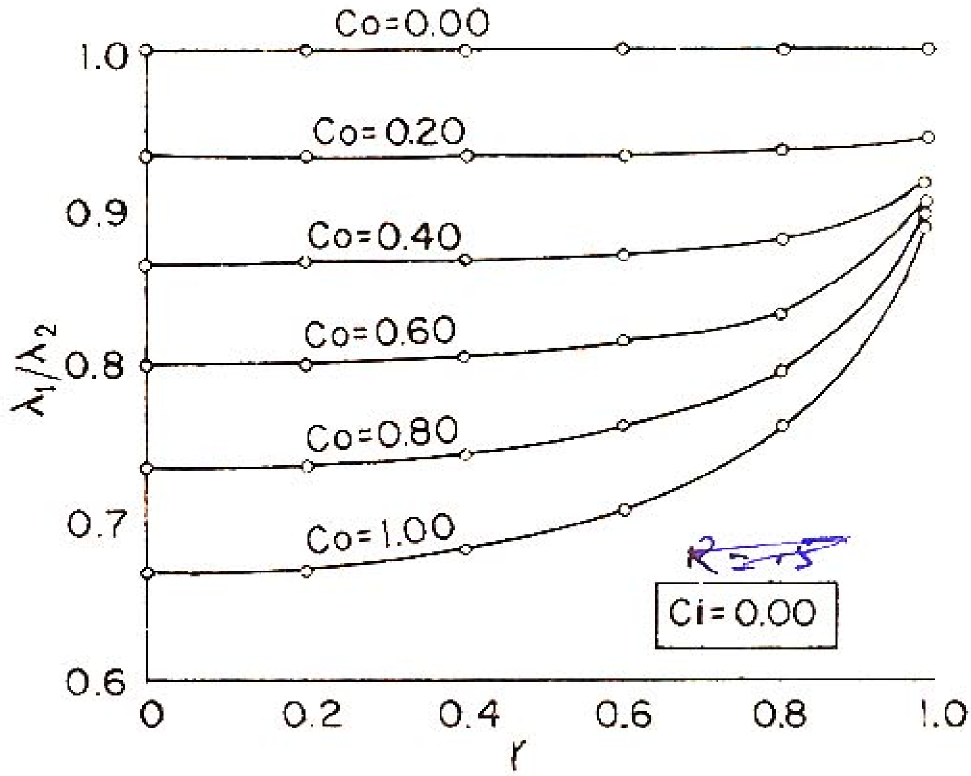
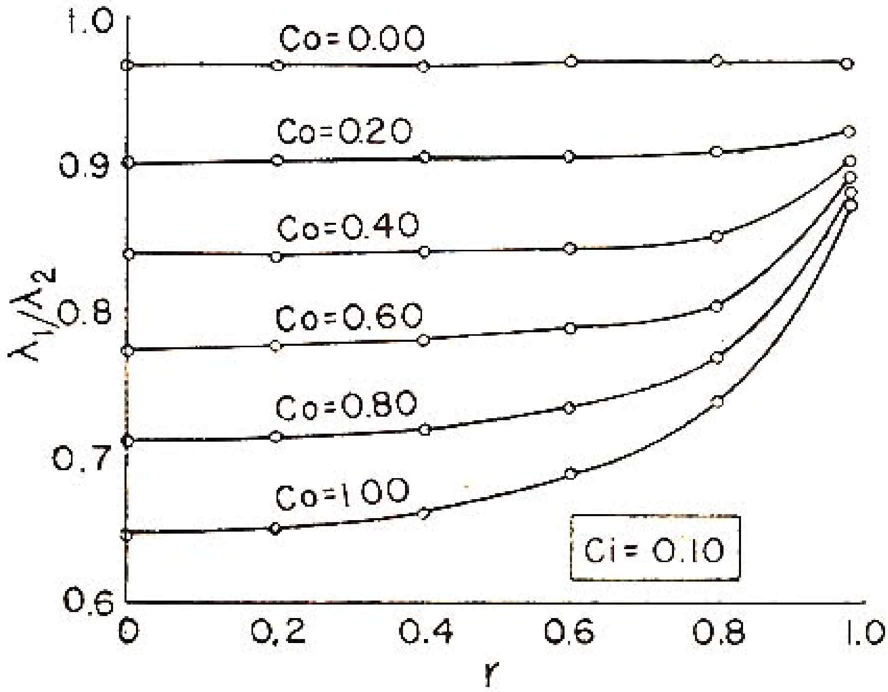


Fig. 4

Curves showing analytical values of critical submergence ratio for $C_i = 0$



r = Constriction ratio = b/B

b = Width of constriction

B = Width of channel

Co = Coefficient of exitloss

Ci = Coefficient of entryloss

λ_1/λ_2 = Critical submergence ratio

Fig. 5

Curves showing analytical values of critical submergence ratio for $C_i = 0.1$

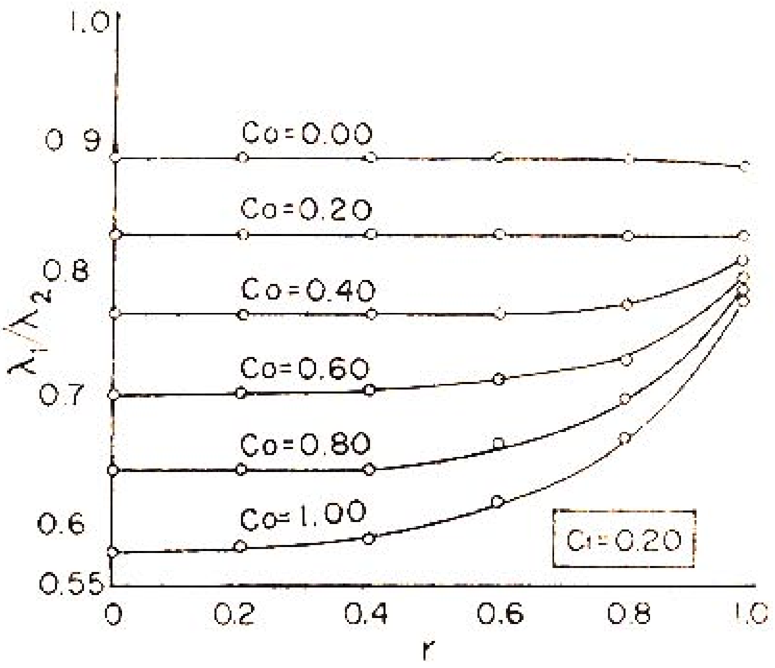


Fig. 6

Curves showing analytic values of critical submer gence ratio for C_i = 0.

r = Constriction ratio = b/B
b = Width of constriction
B = Width of channel
Co=Coefficient of exit loss
Ci= Coefficient of entryloss
λ_1/λ_2=Critcal submergence ratio

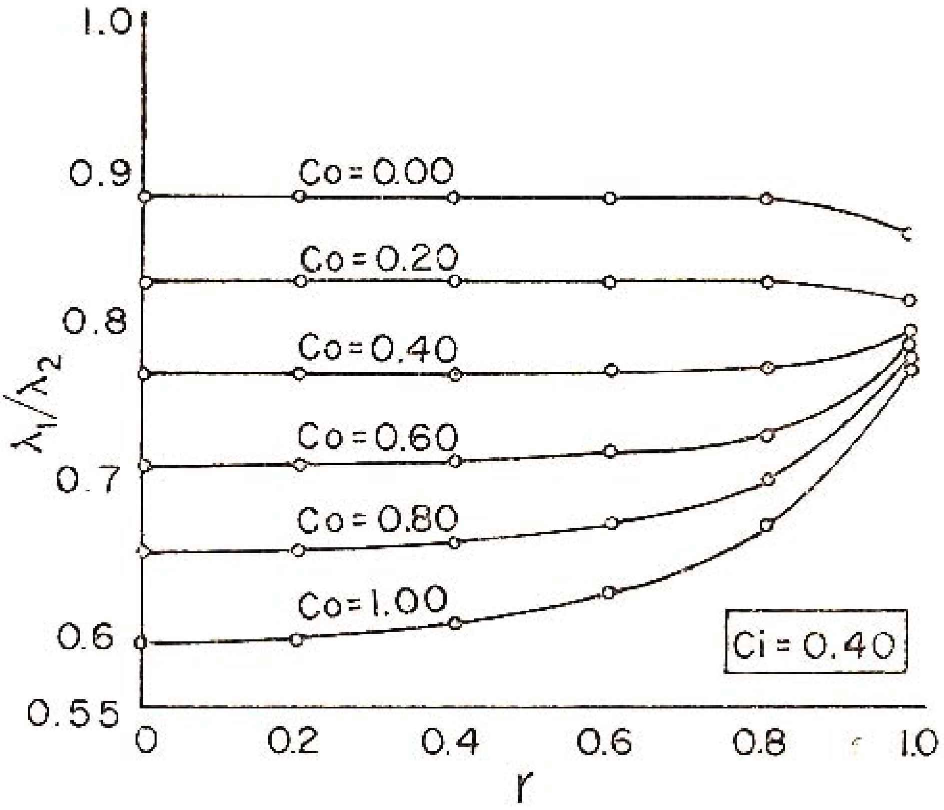


Fig. 7

Curves showing analytical values of critical submergence ratio for $C_i = 0.4$

(iii) Discharges $Q = 0.5$, $Q = 1.0$ and $Q = 2.0$ cusec. for each of the values in item (ii) keeping the entry same in all the cases (Fig. 3).

The points of critical submergence were graphically determined, by gradually raising the tail water by an adjustable weir and noting in each stage the corresponding submergence ratio. The point of critical submergence, as is apparent from Figs. 8 to 10, could easily be located at the junction of the horizontal through Y_1 to the rising limb.

The summary of the observations made and the experimental values of critical submergence ratios are given in Table 2. In Table 3, the values of C_i and C_o, for the various transitions adopted at entry and exit have been calculated and corresponding analytical value of critical submergence ratios indicated. A comparison between the experimental and analytical values are shown in Table 4, in which Balloffet's values as determined from his equation [equation (2)] are also incorporated.

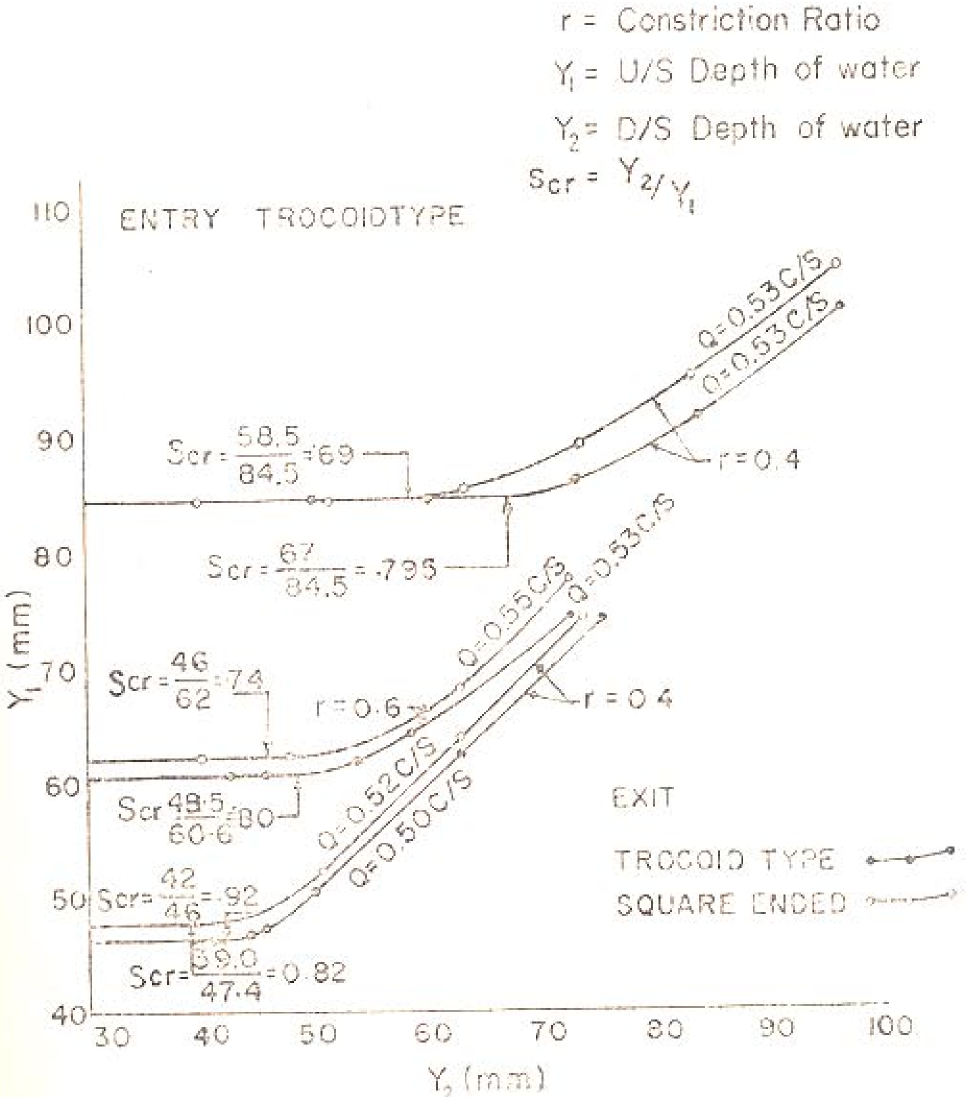


Fig. 8

Curves showing experimental values of critical submergence ratio

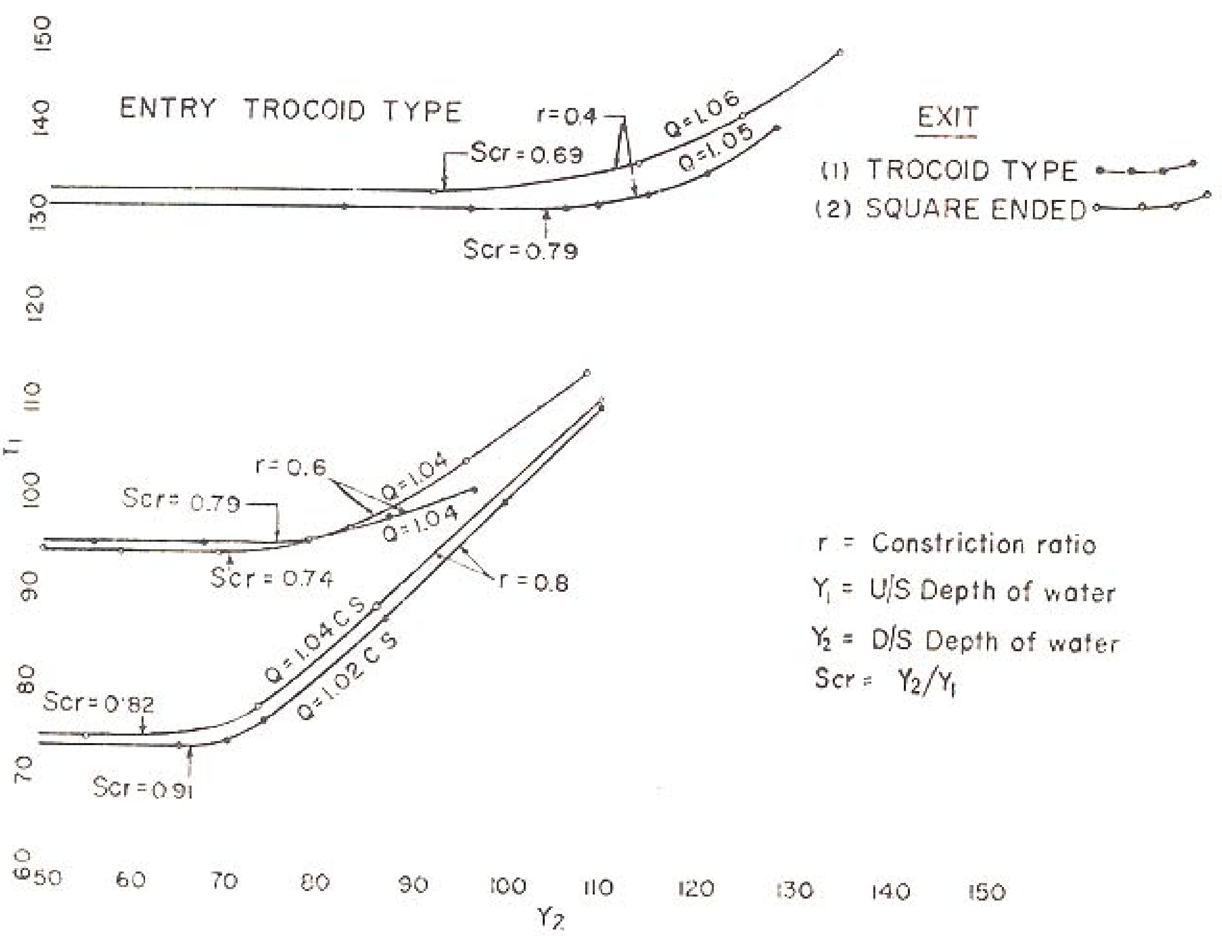


Fig. 9

Curves showing experimental values of critical submergence ratio

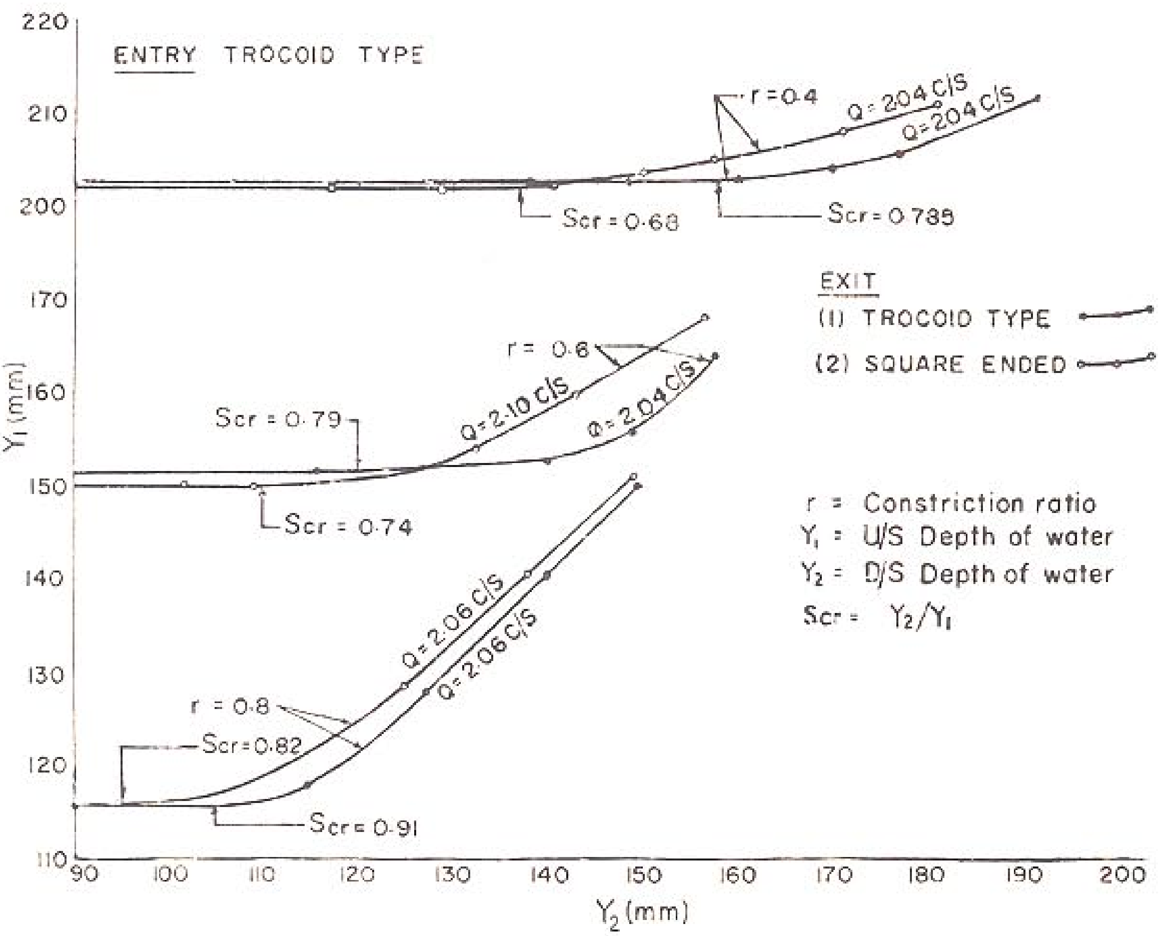


Fig. 10

Curves showing experimental values of critical submergence ratio

Table 2

Summary of experimental observation

r	Entry	Exit	Q, cusec.	Y_1, mm.	Y_2, mm.	$S_{cr.}$ (experimental)
0.4	Trocoid type	Trocoid type	2.04	202.5	158.0	0.785
			1.05	130.8	103.0	0.790
			0.53	84.5	67.0	0.795
	Trocoid type	Square ended type	2.04	202.0	137.0	0.680
			1.06	132.8	92.0	0.690
			0.53	84.5	58.5	0.690
0.6	Trocoid type	Trocoid type	2.04	151.5	120.0	0.790
			1.04	95.0	74.5	0.790
			0.53	60.6	48.5	0.800
	Trocoid type	Square ended type	2.10	150.0	110.0	0.740
			1.04	94.0	70.0	0.740
			0.55	62.0	46.0	0.740
0 8	Trocoid type	Trocoid type	2.06	114.5	105.0	0.910
			1.02	73.0	66.0	0.910
			0.50	46.0	42.0	0.920
	Trocoid type	Square ended type	2.06	115.5	95.0	0.820
			1.04	74.0	61.0	0.820
			0.52	47.4	39.0	0.820

Table 3

Coefficients for entry and exit losses and corresponding critical submergence ratio

(B = 3 ft.)

r	Entry	Exit	Q cusec.	Y_c, ft.	Y_1, ft.	Y_2, ft.	[C_i from equation (8)]	[C_o from equation (14)]	$S_{cr.}$ (analytical)
0.4	Trocoid type	Trocoid type	2.04	0.448	0.664	0.518	0.04	0.646	0.780
			1.04	0.288	0.430	0.338	0.06	0.607	0.787
			0.53	0.182	0.277	0.220	0.12	0.530	0.791
	Trocoid type	Square ended type	2.04	0.448	0.663	0.449	0.04	0.995	0.672
			1.06	0.289	0.434	0.302	0.08	0.895	0.692
			0.53	0.182	0.277	0.192	0.12	0.872	0.689
0.6	Trocoid type	Trocoid type	2.04	0.342	0.495	0.394	0.08	0.583	0.786
			1.06	0.218	0.311	0.244	0.04	0.664	0.784
			0.53	0.139	0.198	0.159	0.04	0.562	0.815
	Trocoid type	Square ended type	2.10	0.348	0.492	0.361	0.01	0.888	0.733
			1.04	0.218	0.308	0.230	0.01	0.838	0.746
			0.55	0.143	0.202	0.151	0.01	0.836	0.746
0.8	Trocoid type	Trocoid type	2.06	0.284	0.378	0.344	0.03	0.260	0.906
			1.02	0.177	0.239	0.216	0.09	0.220	0.901
			0.50	0.111	0.151	0.138	0.13	0.155	0.913
	Trocoid type	Square ended type	2.06	0.284	0.378	0.311	0.03	0.590	0.824
			1.04	0.180	0.243	0.200	0.08	0.537	0.822
			0.52	0.113	0.156	0.128	0.15	0.464	0.819

Table 4

Comparison between experimental and analytical values of submergence ratio

r	Entry	Exit	Q, cusec.	$S_{cr.}$ experimental (from Table 2)	$S_{cr.}$ analytical (from Table 3)	$S_{cr.}$ [Ballofet's equation (2)]
0.4	Trocoid type	Trocoid type	2.04	0.785	0.780	0.905
			1.05	0.790	0.787	0.905
			0.53	0.795	0.791	0.905
	Trocoid type	Square ended type	2.04	0.680	0.672	0.905
			1.06	0.690	0.692	0.905
			0.53	0.690	0.689	0.905
0.6	Trocoid type	Trocoid type	2.04	0.790	0.786	0.895
			1.04	0.790	0.784	0.895
			0.53	0.800	0.815	0.895
	Trocoid type	Square ended type	2.10	0.740	0.733	0.895
			1.04	0.740	0.746	0.895
			0.55	0.740	0.746	0.895
0.8	Trocoid type	Trocoid type	2.06	0.910	0.906	0.916
			1.02	0.910	0.901	0.916
			0.50	0.920	0.913	0.916
	Trocoid type	Square ended type	2.06	0.820	0.824	0.916
			1.04	0.820	0.822	0.916
			0.52	0.820	0.819	0.916

9. Conclusions

The 'limit of submergence' in a 'critical flowmeter' is dependent upon the head losses in entry, exit and friction. Loss in standing wave has no bearing with the limiting submergence condition. Knowing the types of transition used at entry and exit, and the corresponding coefficient of losses, the point of critical submergence, for the given constriction ratio can be directly read from the particular curves applicable. Neglecting friction loss, the actual value of submergence to be adopted for designing the meter is a little less than the theoretical value, which on multiplication with a constant will give the correct value of critical submergence.

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11. References

1. A. Balloffet. 'Critical Flowmeters (Venturi Flumes)'. *Proceedings of the American Society of Civil Engineers*, vol. 81, July 1955, paper no. 743.

2. R. L. Parshall. 'The Improved Venturi Flume'. *Proceedings of the American Society of Civil Engineers*, vol. 89, 1926, paper no. 1956.

3. J. Hinds Discussion on 'The Improved Venturi Flume' by R. L. Parshall. *Proceedings of the American Society of Civil Engineers*, vol. 89, 1926, p. 859.

4. H. Addison. 'Hydraulic Measurements'. *Chapman & Hall Ltd.*, 2nd edition 1949.

5. J. R. Villemonte and V. W. Gunaji. 'Equation for Submerged Sharp Crested Weirs Found Applicable to 6-in. Parshall Flume'. *Civil Engineering*, vol. 23, no. 6, June 1953, p. 62.