

OPTIMUM LENGTH OF TRANSITION IN OPEN CHANNEL EXPANSIVE SUBCRITICAL FLOW*

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Summary

The efficiencies of transitions for various lengths, discharges and Froude numbers of flow have been found with a view to arrive at an optimum length of transition. The velocity distributions at inlet and outlet of the transitions were taken and the energy correction factors were determined with the help of computer. The water surface profiles were gauged and separation zones were demarcated. The results are plotted graphically and some observations are made.

Notations

- A_1, A_2 = cross-sectional areas of flow at inlet and outlet respectively,
 B_x = width of channel at any distance x away from inlet (measured axially),
 B_1, B_2 = widths of channel at inlet and outlet respectively,
 C = numerical constant,
 e = expansion ratio and energy correction factor,
 F_r = Froude number of flow,
 F_1 = Froude number of flow at entry,
 g = acceleration due to gravity,
 H_L = head loss in transition,
 K = constant,
 L = length of transition, *i.e.*, the axial distance between the inlet and outlet points,
 m = momentum correction factor,
 n = constant,
 Q = total discharge,
 R = hydraulic mean depth,
 R_e = Reynolds number of flow,
 V = velocity of flow through an elementary area dA ,
 V_1, V_2 = average velocity of flow at inlet and outlet sections respectively,

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Y_1, Y_2 = depths of flow at inlet and outlet respectively,

α = divergence angle,

α_1, α_2 = energy correction factors at inlet and outlet respectively,

η = efficiency of transition,

Ψ = angle defining transition length such that $\tan \Psi = \frac{B_2 - B_1}{2L}$, and

ρ = density of water.

1. Introduction

There are numerous hydraulic structures, e.g., flumes, syphons, aqueducts, weirs, falls, barrages, etc., where expansive transition is adopted for guiding the water smoothly from the constricted narrow section to the normal wider section. Various types of shapes with varying lengths have been adopted with a view to improving transition performance; but still the present practice of transition design is based mostly on assumptions. One such assumption is regarding the length of transition. The side splay (flaring) governing the overall length of transition is found to vary from 1 in 1 to even 1 in 10.

It is not known exactly as to how the efficiency of a transition varies with lengths and that what are the other governing variables, such as discharge, Froude number of flow, breadth, etc.

2. Historical background

The studies were made in the past by various authors on the problem of sub-critical flow in both expansive and contracting transitions. Most of them are pertaining to closed conduits and relatively less work was done on open channel transitions.

Gibson¹ tried to find out the minimum head loss for his conical diffuser by measuring the values of K in the head loss formula $H_L = \frac{K(V_1 - V_2)^2}{2g}$. The minimum loss was found to occur at a splay of 1 in 16.

Hinds² presented a summary of rules which were established for the design of transition structures by the United States Bureau of Reclamation (U.S.B.R.). The method of design for warped type transition is based on several assumptions, most important one of them being regarding the length of transition which is fixed by an angle of 12°30' between the axis of channel and the line joining the points where the water surface meets the side wall at entry and exit. Other popular types of transitions used by the Bureau are: (i) cylinder-quadrant type, and (ii) wedge type.

The Central Board of Irrigation and Power³, India (C.B.I.P.) made some analytical studies on open-channel expansions with a view to arrive at an optimum rate of divergence. The final expression for the divergence angle α , was found to be

$$\tan \alpha = \frac{C \left(\frac{R}{B}\right)^n + .018 \left(\frac{R}{B}\right)^{-1}}{R_e^{1.5} \times \frac{e}{1 - e F_r} + m \left(2 - \frac{r}{r - 1} + e F_r\right)}$$

where B is the width of the channel, e the energy-correction factor, r the expansion ratio, and m the momentum-correction factor,

All the parameters are for downstream channel, where the flow is least stable. The model studies become essential to determine the more significant parameters which influence the transition's performance predominantly.

It also outlines³ the simplified procedure for transition's design, based on specific-energy principles. But in all the designs, the illustrated lengths of transition have been arbitrarily fixed by splays varying between 1 in 2 to 1 in 3.

Ippen⁴ adopted similar procedure as the C.B.I.P. and made some interesting study of specific-energy curves.

The works done by Mitra,⁵ Chaturvedi,⁶ the Poondi Research Station, Madras,⁷ were all directed towards finding an efficient shape of expansion (the length being assumed arbitrarily).

Kalinske,⁸ Chaturvedi⁹ and some others studied the flow characteristics in an expansion.

Tults¹⁰ conducted a series of experiments in rectangular closed conduits with straight linear boundary. From the results of his experiments, Tults derived the expression:

$$\alpha_{opt.} = \alpha_1 + \alpha_2 \eta \left(1 - \frac{1}{e^3}\right)$$

where $\alpha_{opt.}$ is the optimum rate of divergence for highest recovery of head, η the maximum pressure efficiency, e the expansion ratio, and α_1 and α_2 are the constants. The optimum angle for infinite expansion was found as 8° to the axis.

Tults did not consider non-uniform velocity distribution and ignored the downstream kinetic energy in determining the values of efficiency. Moreover, the results of his experiments conducted in a semi-model closed conduit may not tally with those of a double sided open channel expansion, where resistance varies from point-to-point in the cross-section. This signifies the need for the present investigation.

3. Flow characteristics through expansive transition

The flow through a transition is non-uniform. Although rapidly varied in nature, the flow is treated as gradually varied for calculation of losses, etc. In one-dimensional method of analysis of flow through an expansion, it is assumed that hydrostatic distribution of pressure along and across the channel takes place. The flow is treated as irrotational with uniform distribution of velocity across any section. Such idealized assumptions may, to some extent, be valid only if the rate of expansion is very slow. But in all the practical constructions, where the length of transition has to be limited to some economic amount, neither the pressure distribution is hydrostatic (due to curvilinearity of flow) nor is the velocity distribution uniform (due to varying resistance offered at boundaries). The flow separates invariably in all cases from the boundary, due to adverse pressure gradient and resistance offered at bed and sides of the channel.

Actual flow in a transition is three-dimensional involving various parameters. Due to superimposition of new boundary condition as a result of separation, analytical treatment is difficult for such flows and model studies are generally made. The open channel models are usually constructed on the basis of Froude's criteria, gravity being the predominant force. Under this condition, Reynolds number in model becomes less than that in prototype. Care has to be taken to see that the flow in model is turbulent at least. In prediction of model's results, one has to keep in mind that separation will be less in prototype and consequently the losses too. This is due to the fact that with greater turbulence, there is greater exchange of fluid particles from the slowly moving laminar sub-layer and separation is delayed thereby.

4. Efficiency of transition

In evaluating the efficiency of a transition, it is necessary to consider the velocity variation across the channel. Referring to section 1-1 at the inlet (Fig. 1), let V be the velocity of flow through an infinitesimal area dA . Assuming that the flow is essentially in the direction of the axis of the transition, the volume of fluid per unit time passing through the elementary area is $V dA$. The mass rate of flow through this area is $\rho V dA$. The kinetic energy of this fluid mass per unit time is $\frac{1}{2} (\rho V dA) V^2$, or $\frac{1}{2} \rho V^3 dA$. The total kinetic energy per unit time passing section 1-1 is the integration over the entire area and is represented by: $\frac{1}{2} \rho \int_{A_1} V^3 dA$.

where the subscript A_1 indicates integration over the cross-section 1-1. In a similar manner, the kinetic energy per unit time for section 2-2 is $\frac{1}{2} \rho \int_{A_2} V^3 dA$.

As the fluid passes through the transition, the actual reduction in kinetic energy per unit time, or the power available for transformation in the transition, is the difference of the above two, i.e.,

$$\text{Power for transformation} = \frac{1}{2} \rho \int_{A_1} V^3 dA - \frac{1}{2} \rho \int_{A_2} V^3 dA \quad (1)$$

Not all of this available energy is transformed into useful work. It may be helpful to imagine the transition as a pump. The pump raises the pressure of the fluid entering. Equation (1) might be regarded as the power supplied or input to the pump. The head gained by the fluid flowing through the transition is $(y_2 - y_1)$, where y_1 and y_2 refer to the depths of flow at sections 1-1 and 2-2, respectively. If Q is the rate of flow per sec. potential energy gained by the fluid per sec. will be:

$$\text{Potential energy} = Q \rho g (y_2 - y_1) \quad (2)$$

Equation (2) gives the actual power the pump adds to the fluid. The purpose of transition is to convert kinetic energy into useful pressure energy. The efficiency of the transition is defined by the relationship:

$$\eta = \frac{Q \rho g (y_2 - y_1)}{\frac{1}{2} \rho \int_{A_1} V^3 dA - \frac{1}{2} \rho \int_{A_2} V^3 dA} \quad (3)$$

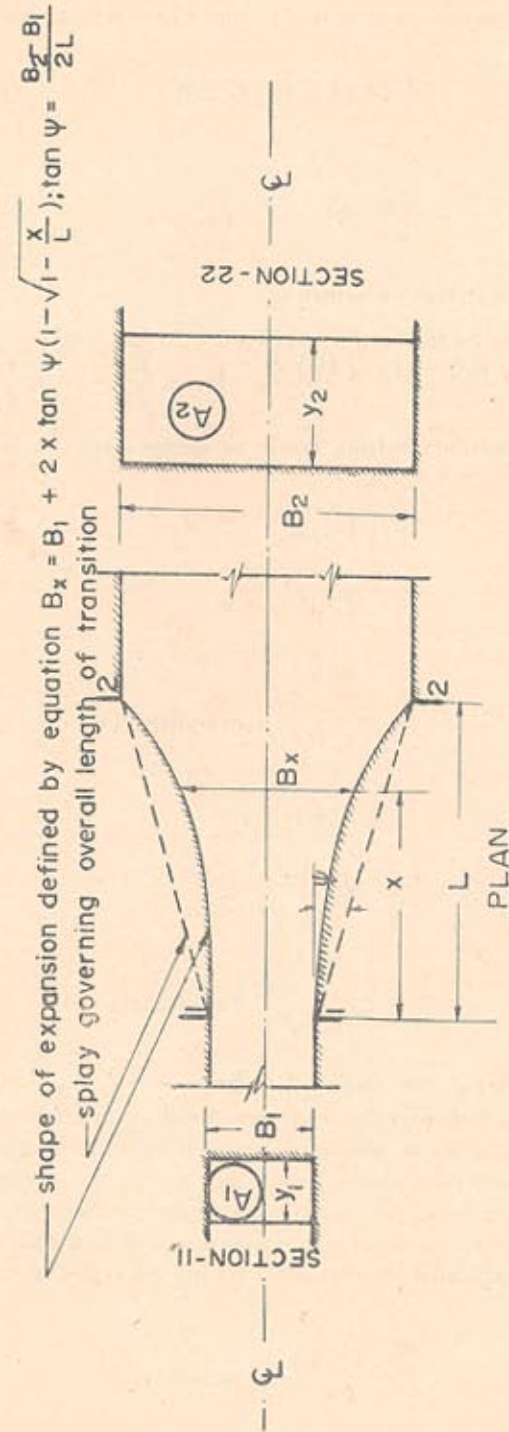


Fig. 1
Typical transition with curved profile

Let V_1 be the average velocity at section 1-1 and V_2 the average velocity at section 2-2. Let,

$$\frac{1}{2} \rho \int_{A_1} V^3 dA = \left(\frac{1}{2} \rho Q V_1^2 \right) \alpha_1 \quad (4)$$

and

$$\frac{1}{2} \rho \int_{A_2} V^3 dA = \left(\frac{1}{2} \rho Q V_2^2 \right) \alpha_2 \quad (5)$$

Equation (3) may, therefore, be written as :

$$\eta = \frac{Q \rho g (y_2 - y_1)}{\alpha_1 \left(\frac{1}{2} \rho Q V_1^2 \right) - \left(\frac{1}{2} \rho Q V_2^2 \right) \alpha_2} = \frac{(y_2 - y_1)}{\left(\times \alpha_1 \frac{V_1^2}{2g} - \times \alpha_2 \frac{V_2^2}{2g} \right)} \quad (6)$$

where, α_1 and α_2 are numerical constants, known as 'energy-correction factors' and are given by :

$$\alpha_1 = \frac{\int_{A_1} V^3 dA}{Q V_1^3} \quad (7)$$

$$= \frac{\int_{A_1} V^3 dA}{A_1 V_1^3}, \text{ from equation (4)}$$

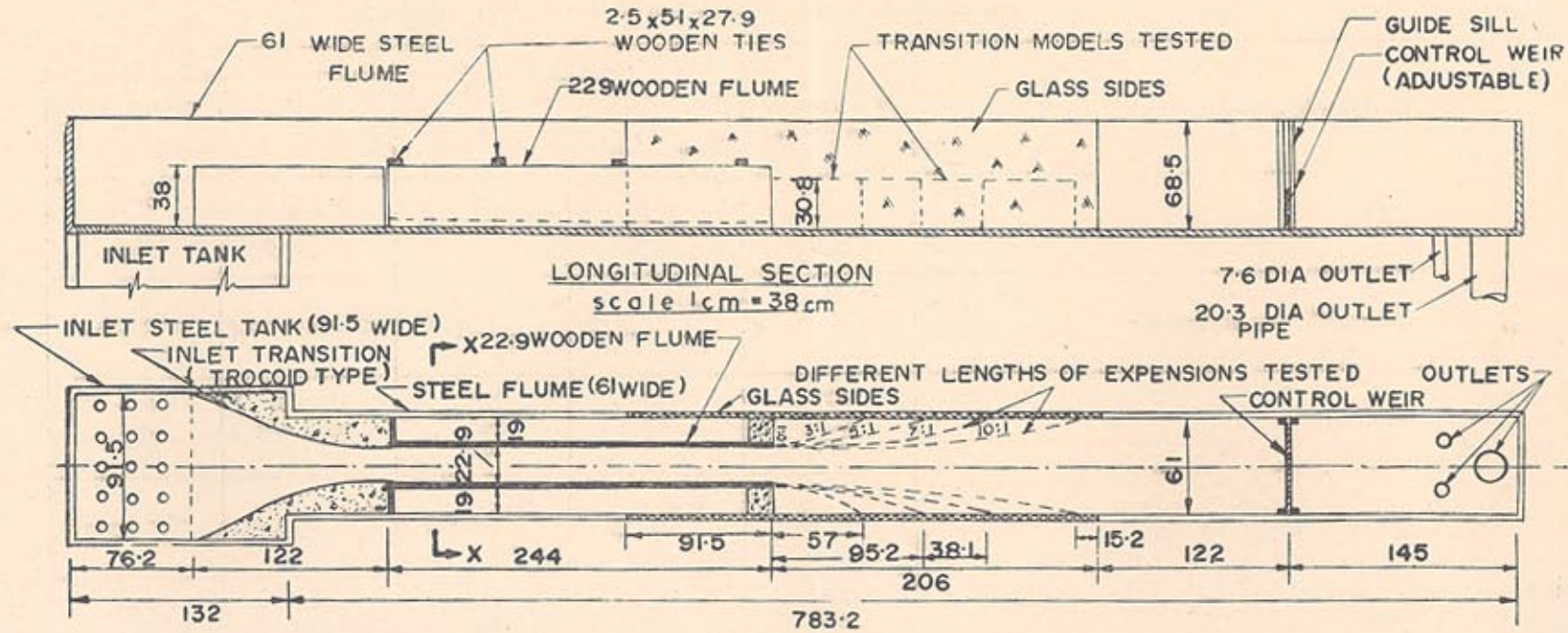
$$\alpha_2 = \frac{\int_{A_2} V^3 dA}{Q V_2^3} \quad (8)$$

$$= \frac{\int_{A_2} V^3 dA}{A_2 V_2^3}, \text{ from equation (5)}$$

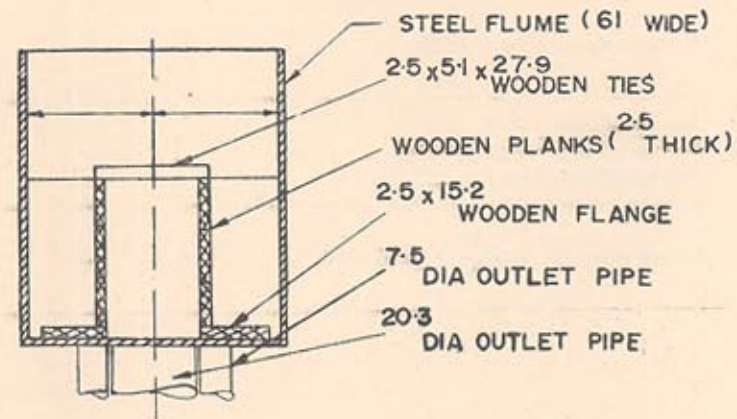
since, $Q = A_1 V_1 = A_2 V_2$, from continuity of flow.

If the velocity is uniform over the initial section and the final section, then $\alpha_1 = 1$ and $\alpha_2 = 1$. The distribution of velocity, however, is never uniform and the values of energy-correction factors are always greater than unity. To determine the exact values of α_1 and α_2 , the velocity contours were plotted for sections 1-1 and 2-2. Annular areas in between consecutive contours were measured by the planimeter. The values of α were then calculated from equations (7) and (8), replacing the integrations by summations as :

$$\alpha_1 = \frac{\sum V^3 dA}{A_1 V_1^3}, \text{ for section 1-1}$$



Note
 All dimensions are in Centimeter



SECTION ON X X
 scale - 1cm = 15.2 cm

Fig. 2

Plan of the flume showing experimental set-up

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Percentage efficiencies of transition for the various conditions of flow

Discharge Q , m. ³ per sec.	Froude number at entry, F_1	Overall length of transition governed by splay as mentioned below				
		0:1 (Abrup expansion)	3:1	5:1	7:1	10:1
0.007	0.75	13.40	56.3	64.0 (71.5)	73.7	65.5
	0.50	12.00	57.7	62.2 (66.0)	70.3	65.0
	0.25	15.95	61.6	67.5 (64.7)	81.8	67.0
Average efficiency for $Q=0.007$	—	$\frac{41.35}{3} = 13.78$	$\frac{175.6}{3} = 58.5$	$\frac{193.7}{3} = 64.6 (67.4)$	$\frac{225.8}{3} = 75.3$	$\frac{197.5}{3} = 65.8$
0.014	0.75	10.4	57.0	52.4 (43)	63.3	39.6
	0.50	8.6	54.8	66.5 (56.0)	79.5	56.8
	0.25	14.0	66.8	80.5 (66.0)	93.5	68.5
Average efficiency for $Q=0.014$	—	$\frac{33.0}{3} = 11.0$	$\frac{178.6}{3} = 59.5$	$\frac{199.4}{3} = 66.5 (55)$	$\frac{236.3}{3} = 78.8$	$\frac{164.9}{3} = 55.0$
0.028	0.75	10.7	49.5	58.7	70.8	77.2
	0.50	7.0	46.0	65.5	65.5	79.5
	0.25	20.4	54.2	68.2	75.7	81.8
Average efficiency for $Q=0.028$	—	$\frac{38.1}{3} = 12.7$	$\frac{149.7}{3} = 49.9$	$\frac{192.4}{3} = 64.1 (69.4)$	$\frac{212.0}{3} = 70.7$	$\frac{238.5}{3} = 79.5$
Average efficiency for $F_1=0.75$	—	11.5	54.3	58.4	69.3	60.8
Average efficiency for $F_1=0.50$	—	9.2	52.8	64.7	71.8	67.1
Average efficiency for $F_1=0.25$	—	16.8	60.9	72.1	83.7	68.5
Overall efficiency (average of grand total)	—	$\frac{112.45}{9} = 12.5$	$\frac{503.9}{9} = 56.0$	$\frac{585.6}{9} = 65.0$	$\frac{674.1}{9} = 74.9$	$\frac{600.9}{9} = 66.8$

$$\alpha_2 = \frac{\sum V^3 dA}{A_2 V_2^3}, \text{ for section 2-2}$$

5. Experiments—set-up and technique

All the experiments were carried out in a flume measuring 9.15m. × 0.61m. × 0.69m. deep, in the Hydraulics Laboratory at the Indian Institute of Technology, Kharagpur. The set-up is shown in Fig. 2. The transition models were constructed with galvanized tin sheets painted with two coats of white enamel paint. The sheets were given required shape by wooden templates at top and bottom connected through wooden struts. Various features of the experiments are described below :

(i) Shape of transition

This represented the boundary of an eddy-zone in an abrupt expansion. The equation to the curve determined experimentally by Lebedev¹² after a number of experiments at Moscow Power Institute, U.S.S.R., is given by :

$$B_x = B_1 + 2x \tan \psi \left(1 - \sqrt{1 - \frac{x}{L}} \right)$$

(ii) Lengths of transition

Five different lengths of transition governed by side splay of 0:1, 3:1, 5:1, 7:1 and 10:1 were used. First one, obviously, represents an abrupt expansion.

(iii) Constriction ratio

All the experiments were conducted in rectangular channel with a constriction ratio of 62.5%, i.e., from a throat width $B_1 = 22.9$ cm. to the channel width $B_2 = 61.0$ cm. cm.

(iv) Discharges

Each length of transition was tested for three different discharges Q as 0.028, 0.014 and 0.007 m.³ per sec.

(v) Froude number of flow

For each discharge, experiments were conducted at three different depths, so that Froude number F_1 , at entry were 0.75, 0.50 and 0.25.

The total number of experimental runs were forty-five. This is apparent from the scheme of experiments as discussed above.

Studies were also made about separation of flow. The points of separation and zones of separation were traced by visual observation with aluminium powder and from velocity distribution.

An in.-gauge measuring correct up to 0.005 in. (0.0127 cm.) was used for determining water surface profile.

Exact distribution of velocity both at the inlet and the outlet of the transitions were taken by 'Prandtl tube'. Velocity at the outlet being very low near the eddy-zone boundary, ordinary water manometer could not be used. Differential manometer with medium oil having specific gravity of 0.823 was used.

6. Results

The percentage efficiencies for different lengths, discharges and Froude number of flow are given in the table. Average values of efficiency for a given discharge or Froude number has also been shown in the table. Overall efficiency of a transition is also given.

The results are plotted graphically in Figs. 3-11, with side's splay as the abscissa and percentage efficiencies as the ordinate. Figs. 3-5 show the variation in efficiency for different values of Froude numbers, keeping discharge the same. The average efficiency (average of Froude numbers) is plotted in Fig. 10. Figs. 7-9 indicate the variation of efficiency for different discharges maintaining same Froude number of flow. Fig. 6 gives the values of average efficiency (average of discharges). The curve of overall efficiency is plotted in Fig. 11, in a semi-log paper, using the results of all the forty-five experiments.

The curve of overall efficiency reveals that the economy obtained initially by adding more length of transition is not available afterwards. When the splay is changed from 0:1 to 3:1, the efficiency increases from 12% to 53%, i.e., an increment of 41%, whereas by bringing about the same amount of change from 3:1 to 6:1, the increment is only 19% (from 53% to 72%). The increment after 6:1 splay is negligible. The highest efficiency attained at about 7:1 splay is found to be 75%. The efficiency falls after 7:1 splay.

Purely from consideration of hydraulic efficiency, therefore, a splay 7:1 may be useful, for the type of boundary tested. But from the practical considerations, e.g., cost of construction, splaying longer than 5:1 or at best 6:1 may not be profitable. A little sacrifice in efficiency may be helpful in overall economy of construction.

Figs. 3-5 show that for the given discharge, the efficiency usually increases with increase in depth. The increment is more in the lower range of Froude number than in the higher ones. The average efficiency's plot (Fig. 6) also reveals the same fact. From Figs. 7-9, it is seen that for given Froude number, the efficiency usually increases with decrease in discharge upto a certain length of transition and the efficiency falls afterwards with decreasing discharge and rises with increasing one. Same conclusions may be drawn from the average efficiency's plot (Fig. 10).

It is also seen from the figures, referred to above that the splay for maximum efficiency varies considerably for different flow conditions. Also, the rate of change in efficiency differs widely for different cases.

7. Observations

Separation

The separation of flow took place at the boundaries near the exit of the transition. Even with a splay of 10:1, it could not be prevented. The points of separation were not symmetrical on either side. It was found that diffusion practically ceased at the point where the livestream re-attached the side, opposite to the eddy-zone. The separation points were found to move forward and backward on either side.

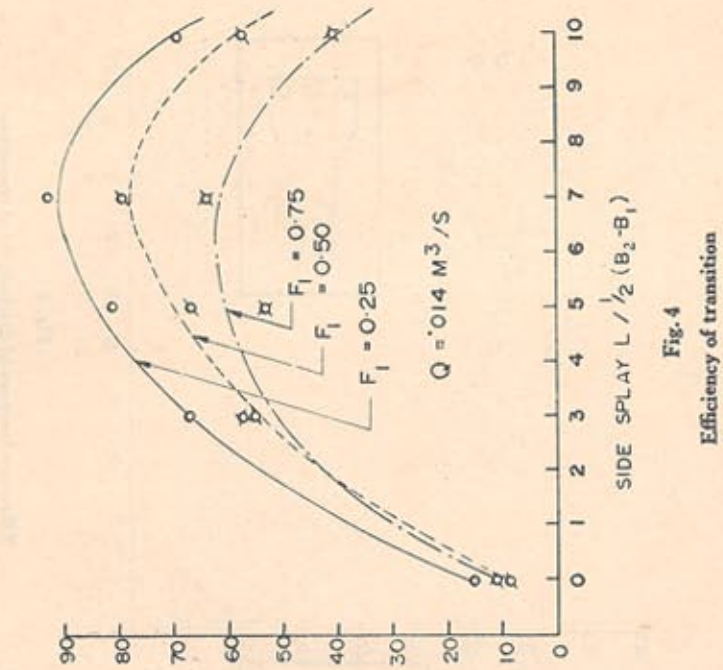


Fig. 4

Efficiency of transition

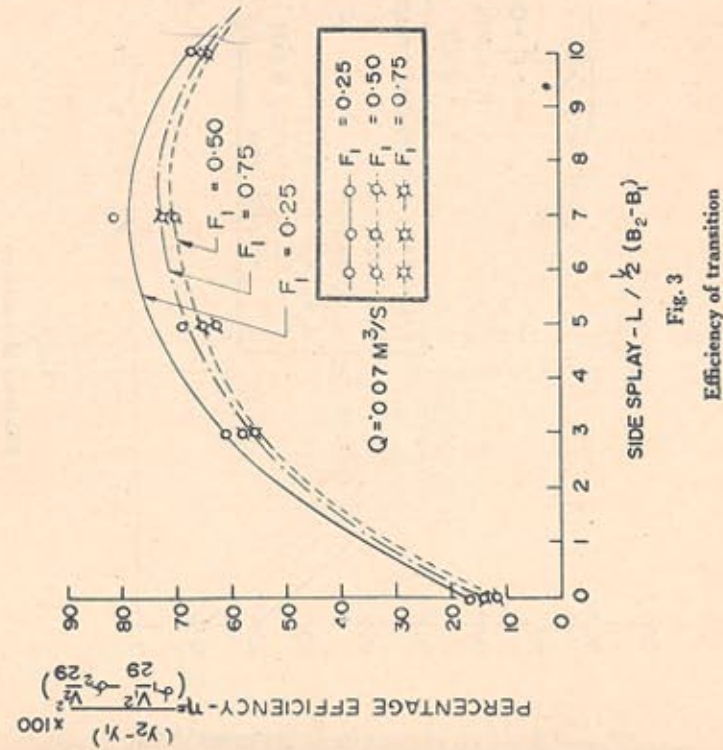


Fig. 3

Efficiency of transition

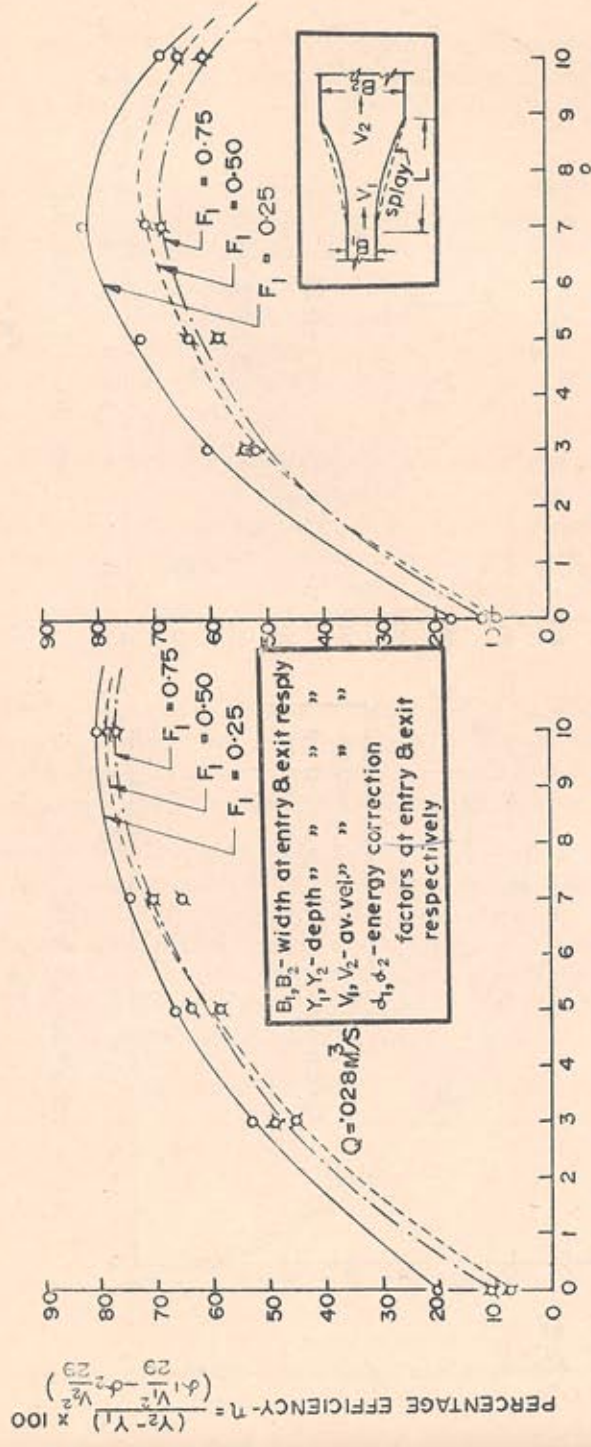


Fig. 6
Efficiency (average of discharges) of transition

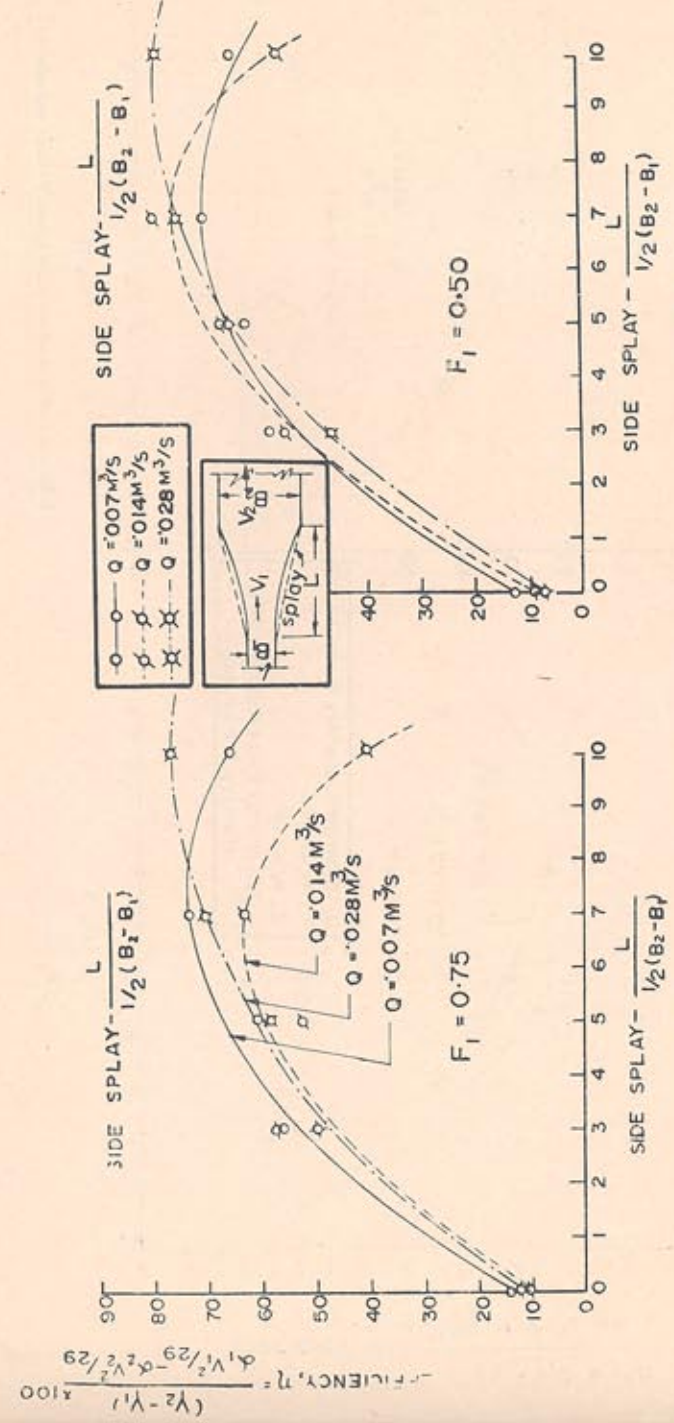


Fig. 8
Efficiency of transition

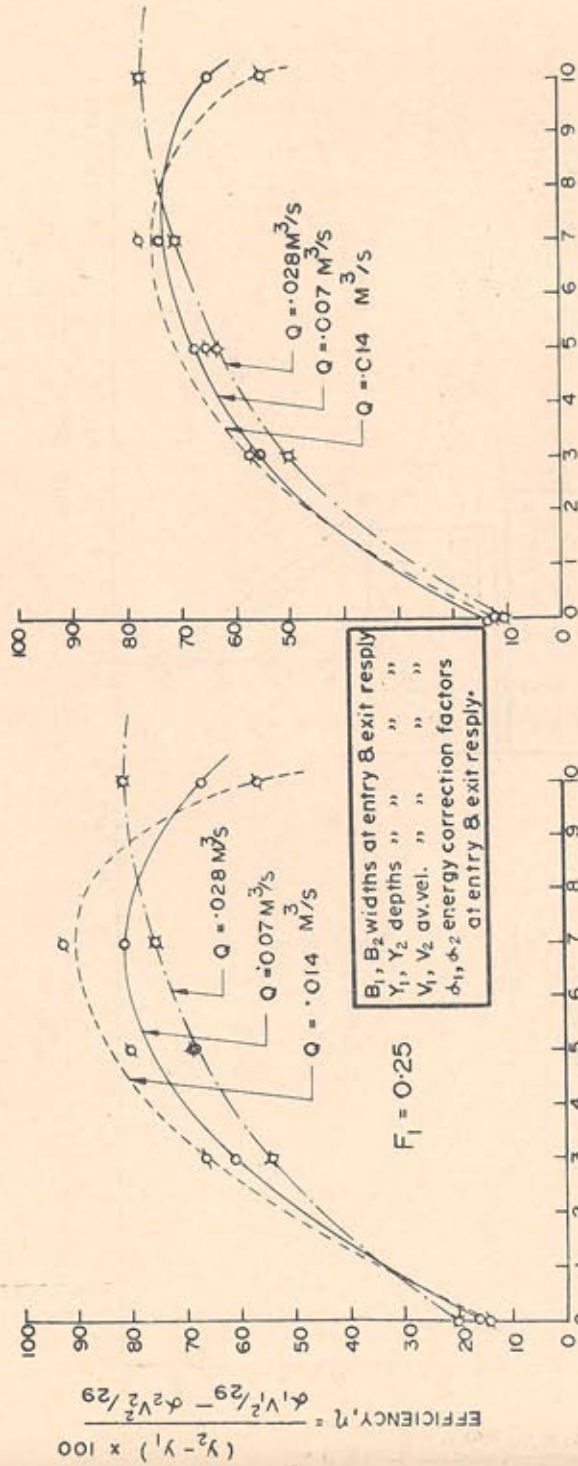


Fig. 9
Efficiency curve

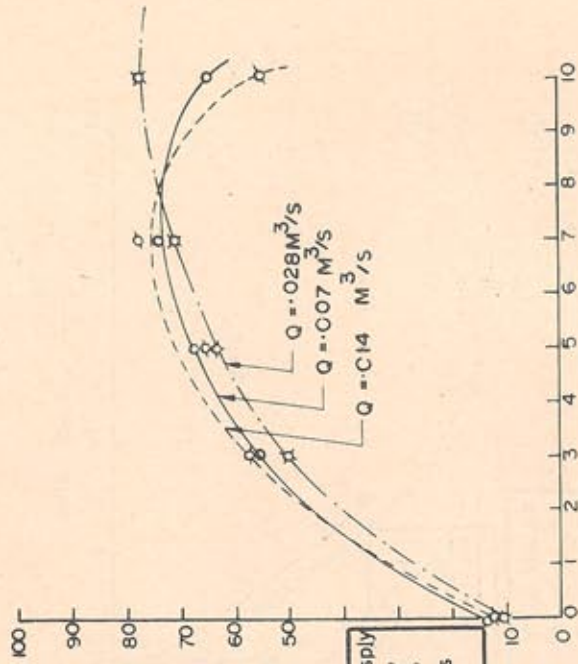


Fig. 10
Efficiency of transition (average of Froude members)

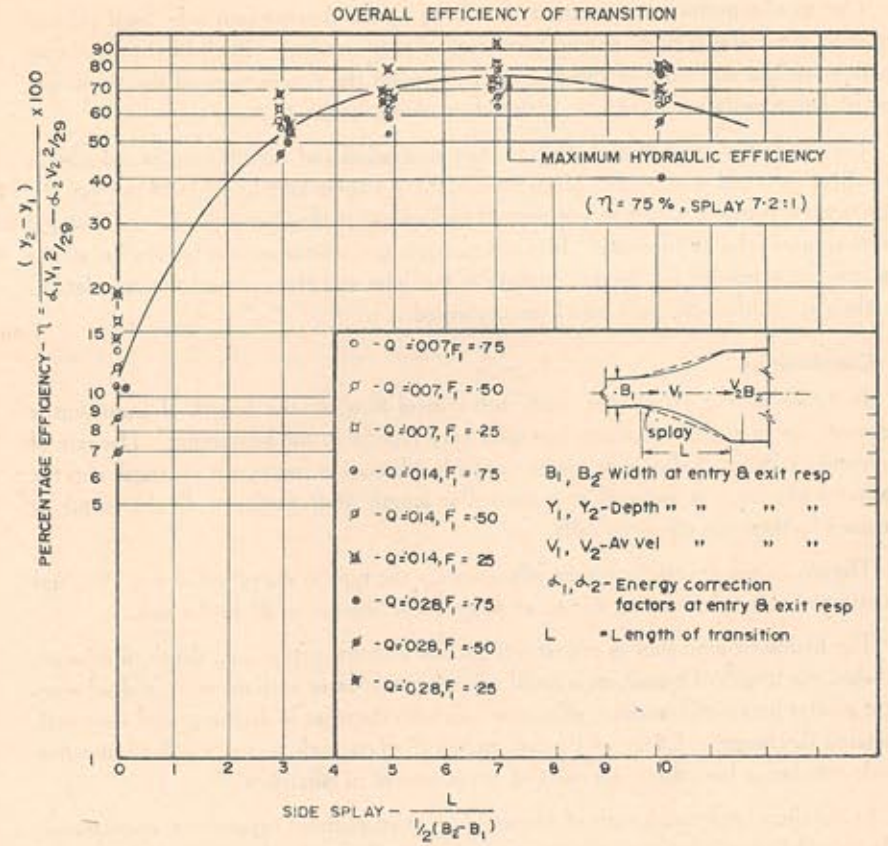


Fig. 11
Overall efficiency curve

Stability

The flow in the downstream channel after expansion was found to be highly unstable and oscillating in nature. At times, flow was found to swing completely from one side to the other, thereby reversing the picture of separation and velocity distribution.

Symmetry

Excepting a few cases, flow was never symmetrical with respect to the centre line of the channel. The maximum-velocity line coincided with the centre line of the channel, for a short length after entry. Thereafter, it shifted to the side to which the main flow attached.

Velocity distribution

The velocity distribution was highly non-uniform after expansion. The non-uniformity of the distribution was indicated by the values of α_2 . Whereas α_1 at the entry ranged from 1.00 to 1.25, α_2 at the exit was found in the range of 2.00 to 4.00. The distribution improved a little by increasing the length of transition.

Due to non-uniform distribution of velocity on the downstream side, local velocity remained considerably high, although average velocity was low. Such localized velocity scours away bed and sides of the channel far beyond the downstream of the transition. The jet being swinging in nature, entire channel is exposed to scour.

Fig. 12 shows the water surface profile, separation and distribution of velocity at the entry, exit and mid-length of transition, for 3:1 splay and $Q = 0.014 \text{ m}^3 \text{ per sec}$. Asymmetry of the flow can be understood by looking at the centre line of channel and the maximum velocity filament. It is not possible to incorporate the figures for all the forty-five experiments. Velocity contours at the inlet and the exit and the calculations for efficiency values, etc. have also been omitted.

8. Conclusions

In open-channel expansions with sub-critical flow, as the length of transition is increased, the hydraulic efficiency increases very rapidly in the beginning. The rate of increment in efficiency, however, decreases with subsequent increment in length and the maximum efficiency is attained at a particular length of transition. If the length is increased further, the efficiency falls.

The maximum overall hydraulic efficiency for the type of curve tested was 75% and was attained at a splay of 1 in 7, i.e., an angle of divergence of 8° to the axis.

The hydraulic efficiency is influenced by flow characteristics, e.g., depth, discharge, etc. when the length of transition is small, efficiency increases with decrease in discharge, but at greater length of transition efficiency falls with decrease in discharge and rises with increasing discharge. Effect of Froude number, when high is not much; but when Froude number is low, there is a marked improvement in efficiency.

In deciding optimum length of transition in open-channel expansions, comparative study should be made between the gain in hydraulic efficiency achieved by increment in length and the loss in revenue due to additional cost of construction arising from such increment in length. Optimum length of transition for higher discharges is greater than for lower ones; but it is independent of Froude number of flow.

The separation can be prevented at a very slow expansion rate which is not economical from points of view of energy's recovery and cost.

The velocity distribution after expansion becomes highly non-uniform with the result that although the average velocity of flow decreases, the local velocity remains considerably high, which if allowed to persist in erodible channels will scour away bed and sides of channel.

In deciding the efficiency of a transition, therefore, a second requirement besides energy recovery is that velocity after expansion should be distributed as uniformly as experienced in normal open-channel flow. In other words, the energy-correction factor for the exit should also be almost the same as at the entry of the transition.

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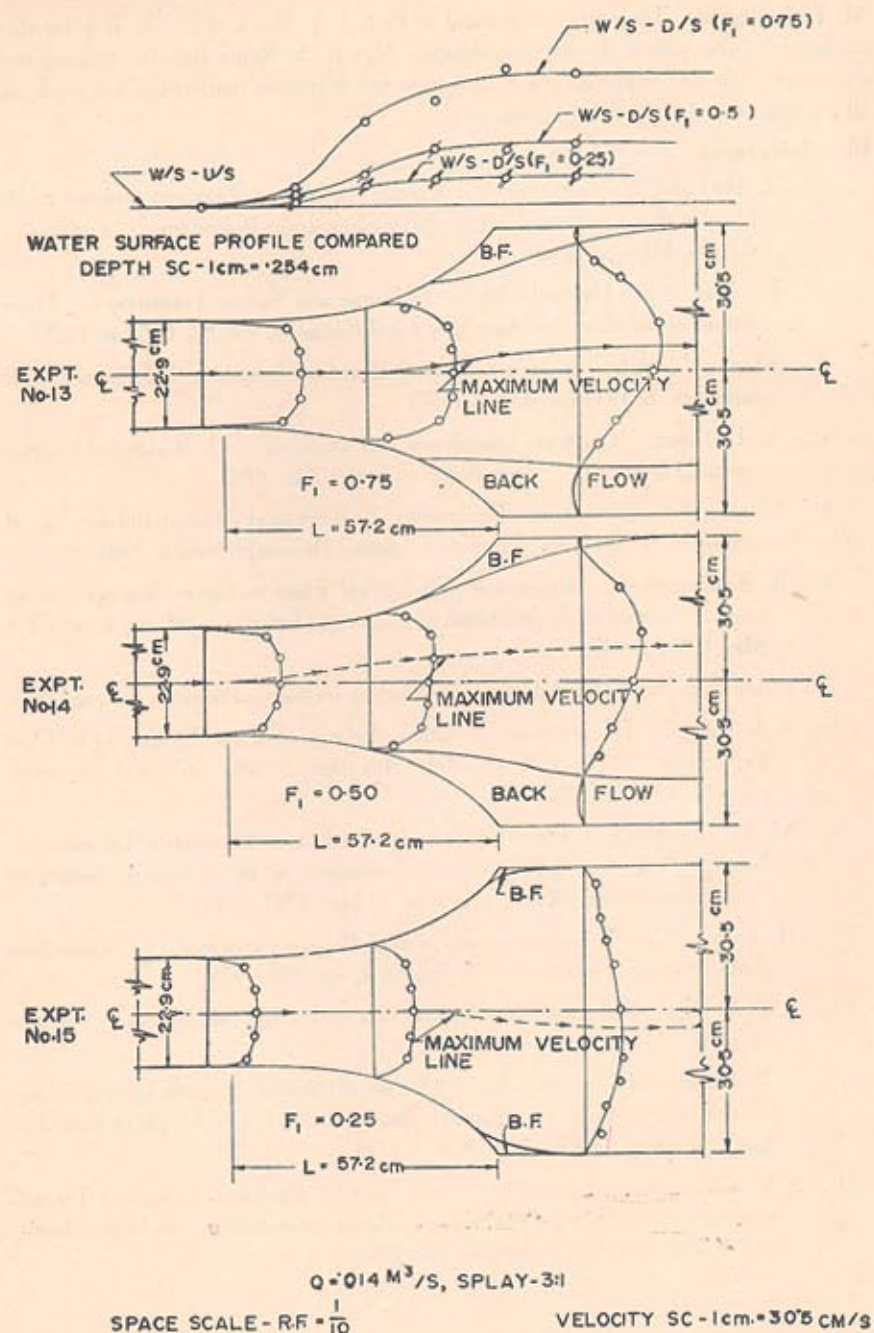


Fig. 12

Plans showing velocity distribution and separation of flow

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