CRITICAL TRACTIVE STRESS IN STRAIGHT AND CURVED CHANELS WITH NON- COHESIVE SOIL

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ABSTRACT

Estimation of critical tractive stress in an open channel is important for finding bank stability and for the design of stable channels. It is also vitally needed for finding bed load transport. Shields' study is applicable for non-cohesive soil on flat surface. Lane made exhaustive study on critical tractive stress for both flat and inclined surfaces and contributed immensely for the design of stable channels. All these studies are, however, applicable for straight channels only. Starting from fundamentals, author has derived equations for critical tractive stresses both for straight and curved channels. It has been shown that the shear stress induced by secondary current in a typical curved channel e.g. in a meandering bend, reduces significantly the critical tractive stress on the outer side of bend. If the stress due to secondary current is very high due to strong flow curvature, critical tractive stress may vanish and the outer bed and bank may cave in even with insignificant forward drag .

KEY WORDS: tractive stress, critical tractive stress, secondary current, bank stability

INTRODUCTION

Critical tractive stress (τ_c) is the shear stress at which the particles on the channel bed and bank are on the verge of motion. In other words, if the actual shear stress on the surface (τ_0) exceeds the critical shear stress, the particles will start moving i.e.

when $\tau_0 < \tau_c$, the surface is stable
when $\tau_0 > \tau_c$, the surface is unstable or in motion

For the design of stable canals/channels, it is important that the surface is stable by ensuring that at no point the actual shear stress exceeds the critical shear stress at the design discharge.

Concept of critical shear stress has been utilized in developing optimum dimensions of a stable channel, every point of whose surface is at critical shear stress. When a channel is in stable state, its top width (T), depth of flow (Y) and its shape (cosine profile) can be worked out theoretically knowing the design discharge, angle of repose (φ) and mean size of soil particles, d_{50} and other properties of the soil. Considerable study has been made by several research workers in the past in determining stable channel dimension (Lacey, 1929, Blench, 1957, Garde and RangaRaju,2000 , Diplus,1990). Critical tractive stress is to be estimated for computing bed load transport. In fact, in almost all bed load equations, τ_c is the most important parameter which must be known since bed load movement occurs only when actual shear stress on bed (τ_0) exceeds critical shear stress (τ_c).
Shields (1936) made exhaustive study to determine critical tractive stress for non-cohesive soil on a flat surface. Fig.1 gives Shields curve in a non-dimensional form. By plotting Shields' function [$\tau* = \tau_c / \{\gamma_s-\gamma) d_s$ against particle size Reynolds number [$R_e^* = u_* d_s / \upsilon$], Shields obtained the function ($\tau*$) for different particle sizes for different flow conditions. Here, u_* is the shear velocity given by $u_* = \sqrt{(\tau_0/\rho)}$, υ is the coefficient of kinematic viscosity of water, ρ is the density of water, d_s is the size of sediment particles, γ is the unit weight of water and γ_s is unit weight of soil.

It may be noticed that $\tau*$ remains constant ($\tau*$=0.056) when $R_e^* \geq 400$. It can be proved that when $\tau* = 0.056$ & $R_e^* = 400$, d_s = 6 mm. For non-cohesive soils on flat surface with $d_s \geq 6$ mm, it can proved that critical tractive stress is give by the relation

$$\tau_c = \gamma d_s/11$$

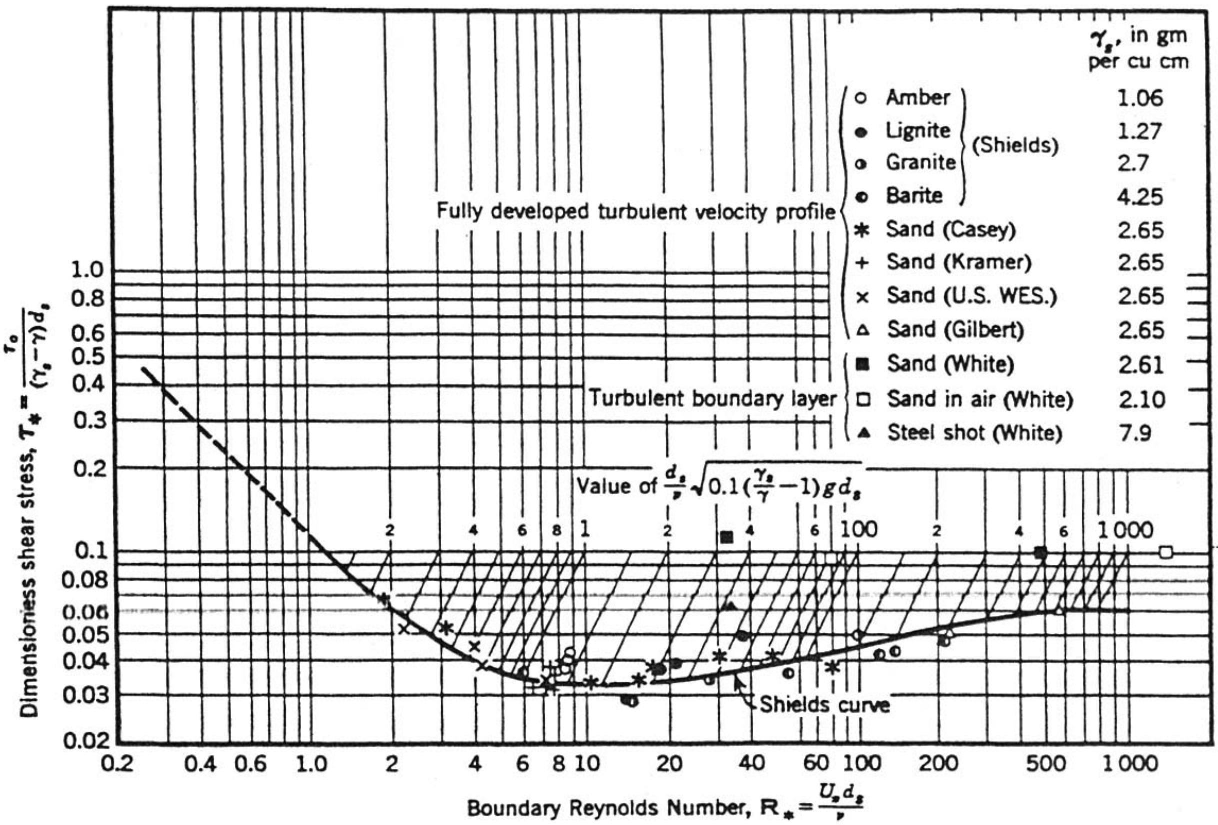

Fig. 1 Shields Diagram for Critical Tractive Stress for Non-Cohesive Soil on Flat Bed

Expressing $\tau_c = \gamma d_s/k_1$, for $d_s \geq 6$ mm, $k_1 = 11$. For sizes less than 6 mm, k_1 – values for d_s less than 6 mm may be obtained from shields' curve (Fig.1) knowing $\tau*$ and R_e^*values at any point on Shields' curve. Subsequently, number of eminent research scientists has conducted important study for finding critical tractive stress from different concept. (White, 1940, Igwaki, 1956, Yalin, 1979).

Shields' work is valid only for uniform and cohesion less sediment particles. For non-uniform cohsion less particles, τ_c- value has been found by Patel and Ranga Raju (1996) and Patel (2006). It may be noted that Shields' curve (Fig.1) is valid for particles lying on flat surface in a straight channel only. For sloping surface, expressions for the critical tractive stress have been determined by Lane (1955), Neil (1968).

In this paper, author has made an effort to determine critical tractive stress for particles on flat and sloping surfaces at the outer bank, when the channel is curved as in a typical meandering bend.

CRITICAL TRACTIVE STRESS IN STRAIGHT CHANNELS

Although τ_c values are well known for flat and sloping surface in a straight channel, author would like to briefly introduce them here for the sake of comparison with the corresponding value in curved channels to be derived in the subsequent paragraphs.

(a) Critical Tractive Stress on Flat Bed Surface (τ_{cb})

Consider a particle 'P' lying on a flat bed (Fig. 2). It will be subjected to a drag (τ_0) in the direction of flow. If the weight of the particles is Ws and the area normal to the flow is a_s, then the drag force on the particle is 'τ_0 a_s', where 'τ_0 'is the shear stress in the flow direction. When the particle is in critical equilibrium (also called threshold condition), $\tau_0 = \tau_{cb}$

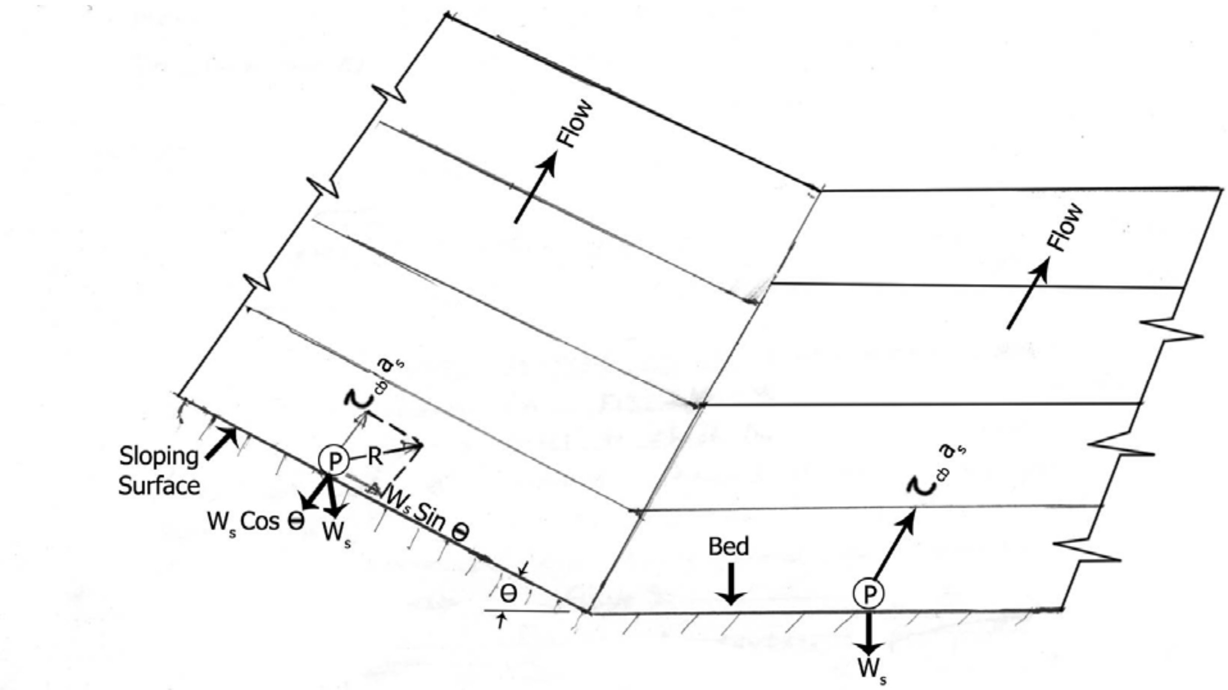

Fig. 2, Showing the various forces on a particle 'P' line on bed and sloping surface in a straight channel.

If τ_{cb} is the critical shear (or tractive) stress on the particle 'P', then the forward drag is given by

$$F_d = \tau_{cb} \,.\, a_s \qquad (1)$$

Resisting force F_R due to its weight W_s acting normal to bed is

$$F_R = W_s \,.\, \tan \varphi \qquad (2)$$

where φ is the angle of internal friction

For equilibrium state, $F_d = F_R$

$$\therefore \tau_{cb}.a_s = W_s \tan \varphi \qquad (3)$$

$$\text{or } \tau_{cb} = W_s \tan \varphi / a_s \qquad (4)$$

Values of τ_{cb} ,as given by Shields, for different sizes of particles, are given in Fig. 1, which may be expressed as

$$\tau_{cb} = \gamma d_s / k_1 \qquad (5)$$

where k_1 is a non-dimensional constant which can be obtained from Shield's curve.(Fig.1). Eq. (5) is non-dimensional; γ is the unit weight of water. Approximate values of k_1 are

for $0.01 < d_s < 3$ mm, k_1 varies from 16 to 60
for $3 < d_s < 6$ mm, k_1 varies from 6 to 11
for $d_s \geq 6$ mm, $k_1 = 11$

(b) **CRITICAL TRACTIVE STRESS ON SLOPING SURFACE (τ_{cs})**

Referring to Fig. 2, consider the particle 'P' lying on sloping surface. The various forces acting on the particle at equilibrium state are: (i)Drag Force, $F_d = \tau_{cs}\, a_s$; (ii) weight component acting in the sloping plane, $W_s \text{Sin}\,\theta$ and (iii) normal component of weight, $W_s \text{Cos}\,\theta$
$\therefore$ Resultant force (R) on the particle trying to dislodge it in the diagonal direction is given by

$$R = [(\tau_{cs} * a_s)^2 + (W_s \text{Sin}\,\theta)^2]^{0.5} \quad (6)$$

Resisting force due to internal friction on the particle is given by

$$F_R = W_s \text{Cos}\,\theta \tan\varphi \quad (7)$$

At equilibrium, $R = F_R$

$$\therefore (\tau_{cs}\ a_s)^2 + (W_s \text{Sin}\,\theta)^2 = (W_s \text{Cos}\theta)^2 \tan^2\varphi \quad (8)$$

or $$(\tau_{cs}\ a_s)^2 = (W_s^2)\,[\text{Cos}^2\theta \tan^2\varphi - \text{Sin}^2\theta] \quad (9)$$

or $$\tau^2_{cs} = W^2_s/a^2_s\,[\text{Cos}^2\theta \tan^2\varphi - \text{Sin}^2\theta] \quad (9a)$$

But from equation 4,

$$(\tau_{cb}\ a_s)^2 = Ws^2 \tan^2\varphi \quad (10)$$

Dividing equation 9 by Eq10

$$\tau_{cs} / \tau_{cb} = [(\text{Cos}^2\theta \tan^2\varphi - \text{Sin}^2\theta)/\tan^2\varphi]^{0.5} \quad (11)$$

on simplification equation 11 may be expressed has

$$\tau_{cs} / \tau_{cb} = [1-(\text{Sin}^2\theta/\text{Sin}^2\varphi)]^{0.5} = k_2$$

$$\therefore \tau_{cs} = k_2\ \tau_{cb} \quad (12)$$

Knowing τ_{cb} from Shields' curve (Fig.1), τ_{cs} can be easily found from equation 12, since k_2 is known for given values of θ and φ. Since k_2 will always be less than unity, the critical tractive stress on a sloping surface will always be less than that on flat bed surface for the same size of particles. When $\theta = \varphi$, $k_2 = 0$ and $\tau_{cs} = 0$, which means that the particle will roll down along the sloping plane even without any drag. Hence, for stability of particles, θ should always be less than φ.

<u>CRITICAL TRACTIVE STRESS (τ'_{CS}) IN CURVED CHANNELS</u>

It is known that when flow occurs in a curved channel e.g. a meandering channel, flow lines are also curved. This induces secondary current in the cross-sectional plane of the channel as shown in Fig.3. It is also established that due to centrifugal forces to which the fluid is subjected to, flow on the outer side of the channel is at higher potential compared to that on the inner side, resulting in the formation of secondary current which moves from the outer bank towards the inner bank as shown in Fig.3 Thus, the soil particles on the bed and bank are subjected to an additional drag (τ_s a_s) in the direction of secondary current as shown in Fig.3. Stability of particle (P) lying on flat and sloping surface in a curved channel is defined in the subsequent paragraphs.

Inner Bank

Outer Bank

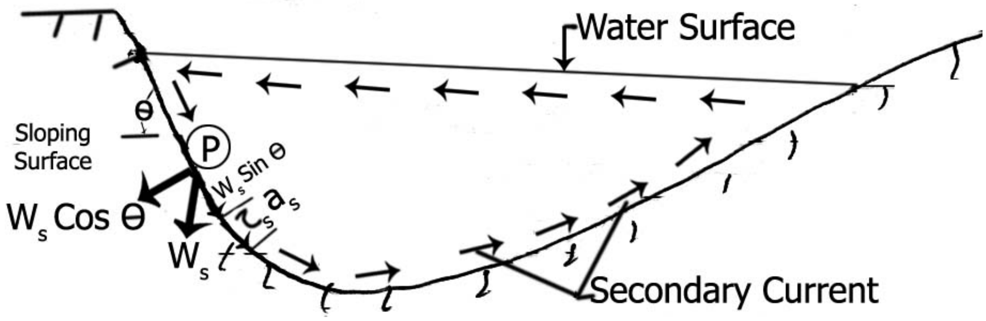

Fig. 3 Showing Secondary Current flowing from Outer to Inner Bank in a Curve Channel.

(a) Flat Surface

Referring to Fig.4, the particle P on the flat bed is subjected to the following forces

(i) Self weight Ws
(ii) Drag force in the direction of flow (τ_0 a_s) which is equal to (τ'_{cb} a_s) under critical state.
(iii) Drag force due to secondary current (τ_s a_s) along the cross-sectional plane as shown in Fig.4.

The resultant force acting on the particle 'P' (Fig.4) on the bed surface is given by

$$R' = [(\tau'_{cb}\, a_s)^2 + (\tau_s\, a_s)^2]^{0.5} \qquad (13)$$

Resisting force on the particle due to its angle of internal friction is given by

$$F_R = W_s \tan\varphi \qquad (14)$$

Under equilibrium condition $R' = F_R$

$$W^2_s \tan^2\varphi = (\tau'_{cb}\, a_s)^2 + (\tau_s\, a_s)^2 \qquad (15)$$

Or, $$\tau'^2_{cb} = (Ws^2 \tan^2\varphi / a_s^2) - \tau_s^2 \qquad (16)$$

From Eq. (4) and (16)

$$\tau'^2_{cb}/\tau_{cb}^2 = 1 - \tau_s^2 (a_s/W_s)^2 \tan^2\varphi \qquad (17)$$

Thus for any finite value of τ_s i.e. shear stress due to secondary current, τ'_{cb} i.e. critical tractive stress on flat bed in a curved channel near the outer bank will always be less than τ_{cb} i.e the critical tractive stress on flat bed in a straight channel..

(b) Sloping Surface

Referring to Fig.4 again, the particle P on the sloping surface is subjected to the following forces.

(i) Component of weight along the inclined plane – W_s Sin θ
(ii) Drag force due to main current in the direction of flow - (τ_0 a_s). Under the equilibrium condition when $\tau_0 = \tau'_{cs}$, drag force in the flow direction - $F_d = \tau'_{cs}\, a_s$
(iii) Drag force due to secondary current - τ_s a_s here τ_s is additional drag on the particle in the cross-sectional plane due to secondary current

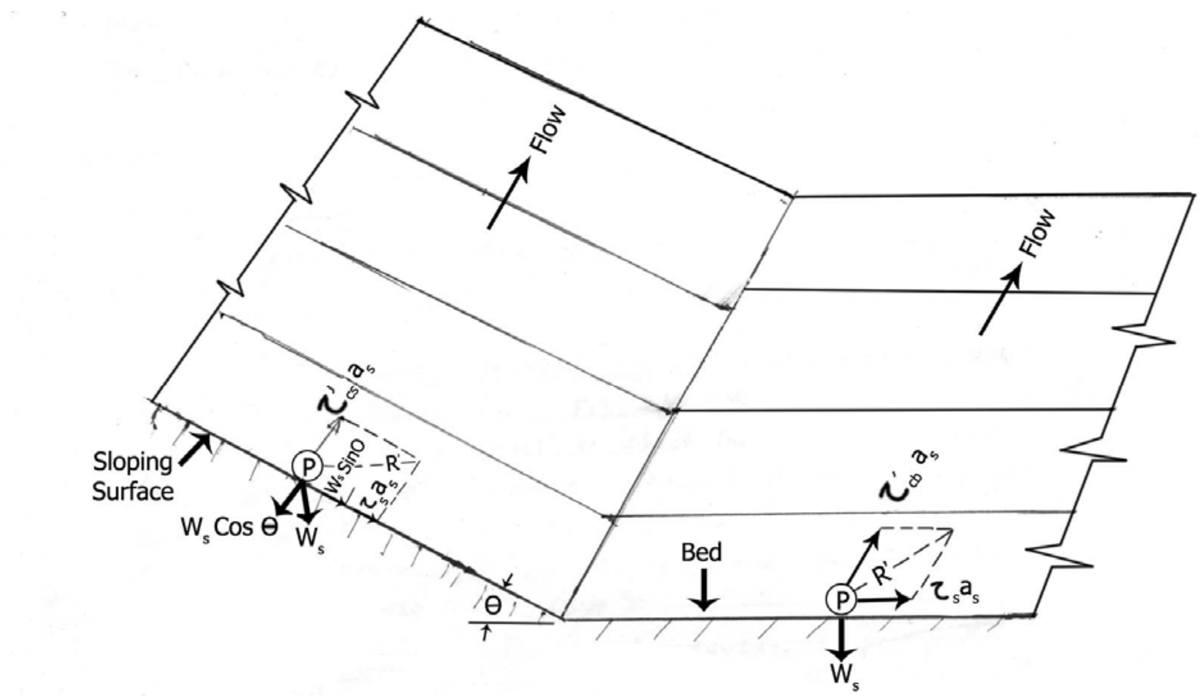

Fig. 4 Showing the Various Forces on a Particle 'P' line on Bed and Sloping Surface in a Curved Channel.

Resultant force ($R^{/}$) is given by

$$R^{/} = [(W_s \text{ Sin } \theta + \tau_s \, a_s)^2 + (\tau^{/}_{cs} \, a_s)^2]^{0.5} \tag{18}$$

Under equilibrium condition,

$$F_d = W_s \text{ Cos } \theta \text{ Tan } \varphi = R^{/} \tag{19}$$

From equation 18 & 19

$$(W_s \text{ Sin } \theta + \tau_s \; a_s)^2 + (\tau^{/}_{cs} \, a_s)^2 = W_s^2 \text{ Cos}^2\theta \tan^2\varphi \tag{20}$$

Or, $$(\tau^{/}_{cs} \, a_s)^2 = W_s^2 \text{ Cos}^2\theta \tan^2\varphi - W_s^2 \text{ Sin}^2\theta) - \tau^2{}_s \, a^2{}_s - 2W_s \, \tau_s \, a_s \text{ Sin } \theta$$

$$\therefore \; \tau^{/}_{cs}{}^2 = [W_s^2 / a_s^2 \, (\text{Cos}^2\theta \tan^2\varphi - \text{Sin}^2\theta)] - \tau_s^2 - 2 \, (W_s/a_s) \, \tau_s \text{ Sin } \theta \tag{21}$$

From Eqs. 9a and 21

$$\tau^{/}_{cs}{}^2 = \tau_{cs}{}^2 - (\tau_s{}^2 + 2 \, \tau_s \, W_s \text{ Sin } \theta \, /a_s) = \tau_{cs}{}^2 - k_3$$

Where

$$k_3 = (\tau_s{}^2 + 2 \, \tau_s \, W_s \text{ Sin } \theta \, /a_s) \tag{22}$$

From equation 22, it is apparent that for any finite value of τ_s , K_3 is positive and hence $\tau^{/}_{cs}$ i.e. critical tractive stress on sloping surface in a curved channel will always be less than τ_{cs} i.e .critical tractive stress on sloping surface in a straight channel without any secondary current. When secondary current (and hence τ_s) is very strong due to sharp curvature of flow lines such that $k_3 = \tau_{cs}{}^2$, $\tau^{/}_{cs}= 0$ i.e. the outer bank will cave in even when the forward drag is zero

SUMMARY AND CONCLUSIONS

Estimation of critical tractive stress is important for finding bank stability in open channels as well as for computation of bed load. Several research scientists have worked on the subject in the past for determining critical tractive stress on cohesion-less soil on flat and sloping surfaces. Shields curve (Fig.1) is very popular for finding critical tractive stress on non-cohesive soil on flat surface. Similarly, Lane has contributed immensely for finding stability of particles on flat and sloping surfaces and in the design of stable channels. However, these studies are applicable for straight channels only. Starting from fundamentals, author has derived expressions for critical tractive stress in both straight and curved channels (as in a typical meandering bend), for the purpose of comparison of the critical stresses in straight and curved channels. It is observed that the shear stress, $\tau_{s,}$ arising out of secondary current has a pronounced effect on critical tractive stress in a curved channel. If the secondary current is very strong, critical tractive stress on the outer side of bends may vanish which implies that the bed and bank materials on the outer side of a meandering bend are highly unstable and they may cave in even at zero or insignificant forward drag..

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