USE OF SMALL BRIDGES AND CULVERTS FOR STREAM GAUGING

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ABSTRACT

There are innumerable small bridges and culverts all over the country. A large number of small bridges and culverts are also going to be constructed for the new roads proposed by the Govt. of India. Small bridges and culverts can be conveniently used for stream gauging. In this paper author has discussed the basic hydraulic principles involved in the design of small bridges and culverts so that they can be used for stream gauging. An example has been worked out at the end.

KEYWORDS

Bridges, Culverts, Critical Flow, Flow Meter, Flow Separation.

1.0 INTRODUCTION

Bridges having span from 6m to 30 m are normally designated as small bridges. Culverts are those having span less than 6m. There are innumerable existing small bridges and culverts and many more are going to be constructed all over the country. These bridges and culverts can be conveniently used for stream gauging. Flow data forms one of the basic requirements for planning and design of bridges and appurtenant works besides other developmental activities.

Stream gauging by conventional methods e.g. area-velocity method are costly since equipment like current meter, ADV, ADCP etc. need money and time, trained personnel and periodic calibration of the instruments. Use of small bridges and culverts for stream gauging is a simple and accurate device since it needs only water level upstream in case flow is free. In case flow is submerged, both upstream and downstream water levels are required. However, some basic understanding of hydraulic principles is essential for accurate determination of flow.

In this paper, author has made an attempt to explain the basic hydraulic principles involved in stream gauging by use of bridges and culverts.

2.0 DIFFERENT METHODS OF STREAM GAUGING

Different conventional methods of stream gauging are available in several text books (Chow, 1973; Rangaraju, 1993; Subramanya, 1985; French, 1986; Mazumder, 2007) and Handbooks of hydraulics (King,1954; Boss,1976). The textbook, 'Weirs and Venturi Flumes' by Ackers et. al (1978) is an excellent text book giving detailed methodology to be adopted for flow measurement by weirs and flumes. Several conferences on 'hydrometry' which have been held in the past are another important source of information for stream gauging. Innovative methods of flow measurement have been published in several Indian journals (e.g. IE (I), ISH, IWRS IWWA, etc.) and foreign journals also (ASCE, IAHR).

3. Hydraulic Principles Involved

For using bridges and culverts for stream gauging, it is necessary to understand some basic hydraulic principles which are discussed in the following paragraphs.

3.1 Critical, Sub-Critical and Super- Critical flow

Flow in open channels is classified as sub-critical and super- critical depending on Froude's number of flow in the channel (F) defined as

$$F=V/\sqrt{(gy)} \qquad (1)$$

where,

V is the mean velocity of flow, y is depth of flow and g is acceleration due to gravity. Flow is critical when F=1 , sub-critical when F<1 and super-critical when F>1. Expressing V=Q/A, Eq. (1) may be written as

$$F= =Q/A\sqrt{(gy)} \qquad (2)$$

For a rectangular channel, A=By where B is the mean width of channel, eq. (2) becomes

$$F= (Q/B)/\sqrt{g}\ y^{3/2} =q/\sqrt{g}\ y^{3/2} \qquad (3)$$

where q is flow intensity i.e. discharge per unit width

In critical flow, F=1 and $y=y_c$ and from eq.(3)

$$y_c=(q^2/g)^{1/3} \qquad (4)$$

where, y_c is the critical depth of flow

In sub-critical flow, F<1 and $y>y_c$. In supercritical flow, F>1 and $y<y_c$

3.2 Specific Energy Principle-Head Discharge Relation

Specific energy is the energy of flowing water measured above river bed i.e. the difference between the total energy level and bed level at any section. If y is the depth of flow and V is the mean velocity of flow, specific energy (E) is given by the relation

$$E=y+V^2/2g= y+Q/2gA^2 \qquad (5)$$

In a prismatic channel, A is a function of y only. For a given Q-value, E is,therefore, a function of y for any given Q-value. Fig.1 illustrates specific energy diagram showing relation between E and y for given Q-values. Differentiating equation (5) with respect to y, it can be shown that when specific energy is minimum, F=1 when $y=y_c$ i.e. the flow is critical as seen in Fig.1. For any given E-value, flow may occur at two alternate depths ($y_1$and y_2) - y_2 greater than y_c and y_1 less than y_c. At point C where specific energy is the minimum, $y=y_c$ for the given flow Q. y_1 and y_2 are the super-critical and sub-critical depths respectively. For rectangular section of width B_o, it can be shown from Eq. (5) that

$$y_c= 2/3\ E_c=(q^2/g)^{1/3} \text{ or } q=(8g/27)^{0.5}\ (E_c)^{3/2}=1.70\ (E_c)^{3/2} \qquad (6)$$

Replacing q by Q/Bo and E_c by H, equation (6) becomes

$$Q=1.70\ B_0H^{3/2} \text{ (in metric unit)} \qquad (7)$$

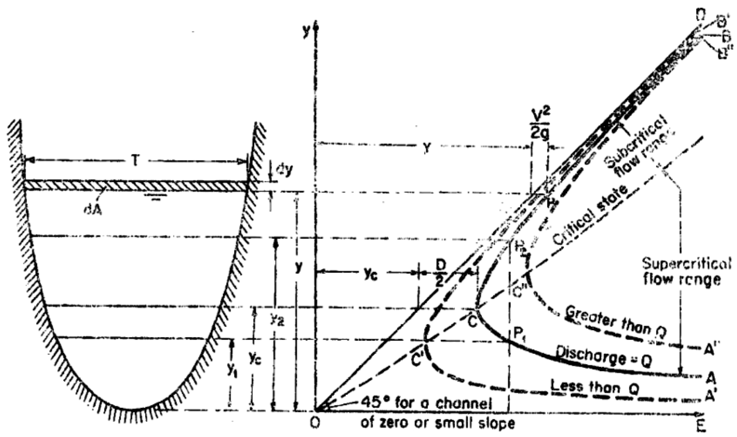

Fig. 1 Specific Energy Diagram

Where, H $=E_c$ i.e. the energy head above crest at critical (or control) section of rectangular width B_0,and q is the discharge intensity (in cumec per meter) i.e. $q=Q/B_0$.

Fig.2 illustrates a typical proportional type flow meter (Mazumder et.al, 1999) where the control section C is developed by simultaneous contraction of channel width from B_1 to B_o and the floor level is raised by an amount Δ above bed such that the flow is just critical state at the control section C (Fig.2). It may be seen that H i.e. head above crest is less than E_c due to loss in head between approach section of width B_1 and the control section C (i.e. h_{Li} in Fig.2). When there is no loss in head i.e. $h_{Li}=0$, equation (7) gives the exact discharge passing through the channel. However, there is always some head loss between approach and control section due to friction and form loss (in case of separation of flow from boundary). Hence, h_{Li} is not zero and equation (7) can be modified as

$$Q=C_d\, B_0 H^{3/2} \qquad (8)$$

where, C_d is the coefficient of discharge. C_d=1.70 $m^{1/2}$/sec where head loss (h_{Li}) is zero. For a given E_1, more the loss of head (h_{Li}), less will be E_c and y_c resulting in lower discharge and smaller C_d-value. C_d-value may vary from 1.50 (for a sudden contraction) to 1.70 (for a venture flume with smooth inlet transition). C_d-value is more than 1.70 due to curvature of water surface profile and negative pressure at control section e.g. ogee type profile in a spillway where C_d-value may be as high as 2.18 $m^{1/2}$/sec.(USBR,1968) .

3.3 Free and Submerged Flow

So long flow passes through critical state or y_c exists at the control section, flow is free and equation (8) can be used for computing discharge. In free flow, discharge is a unique function of upstream water level only and is not affected by downstream water level. When y_0 (at B_o) is greater than y_c , control vanishes and the flow is submerged. Here, y_0 is the depth of flow at control section and $y_0>y_c$.Once the flow is submerged Eq.7 is no more applicable. If the flow is submerged, C_d is affected by both upstream and downstream water levels. Equation (8) can be still used for computing flow with reduced C_d-value. C_d is zero when there is 100% submergence. C_d-values for free and submerged flow conditions are available in the text books cited in section-2.

3.4 Modular Limit /Critical Submergence

Modular limit or critical submergence may be defined as the limiting value of submergence up to which the flow is free and C_d remains more or less constant. It helps in determining whether a given flow is in free or submerged. Defining submergence as $S= y_2/y_1$ and modular limit/critical

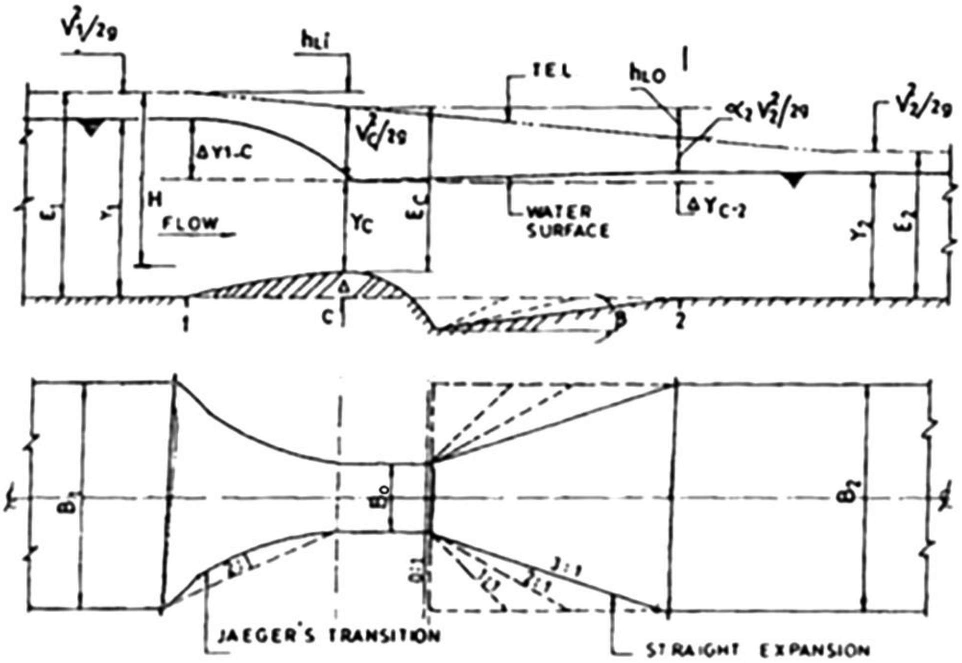

Fig. 2 Showing Plan and Section of a Proportional Flow Meter (Mazumder & Deb Roy, 1999)

submergence as $S_{cr}=(y_2/y_1)_{cr}$, flow is free if $S<S_{cr}$ and submerged if $S>S_{cr}$. Here y_1 and y_2 are upstream and downstream depths of flow as shown in Fig.2. Higher the modular limit, more efficient is the flow meter. Mazumder (1981,1966) proved that modular limit depends on the following parameters (Fig.2):

(i) Fluming or constriction ratio in plan: $r=B_0/B_1$

(ii) Vertical constriction ratio: $R=\Delta/y_1$

(iii) Inlet head loss co-efficient: $C_i=h_{Li}/(V_c^2/2g-V_1^2/2g)$

(iv) Outlet head loss co-efficient:$C_0= h_{L0}//(V_c^2/2g-V_2^2/2g)$

Fig.3 shows the variation of modular limit with C_i ,C_o and r for a given R=0.2. It may be noticed that with rise in head losses (C_i&C_0), modular limit decreases. Modular limit is not significantly affected by r and R. C_i and C_o values are governed by nature of transition at inlet and outlet of flow meter, especially at low values of C_o.

4.0 Proportional Type Flow meter

A flow meter where there is negligible afflux and continues to act under free flow condition irrespective of magnitude of incoming discharge may be termed as a proportional type flow meter. It has an advantage over other classical type flow meters due to the fact that depth-discharge relation can be maintained at all incoming flows and there is no backwater and sediment deposition upstream due to normal flow conditions prevailing at all discharges. Mazumder and Deb Roy (1999) developed the unique flow meter by simultaneous fluming in both horizontal and vertical plain as shown in Fig.2. It acts always under free flow condition irrespective of magnitude of incoming flow in the flow range Q_{max} and Q_{min} used for design of flow meter.. The equations developed for finding the width (B_0) and corresponding rise (Δ) at control section are:

$$Bo = [0.7 (Q_{max}^{2/3} – Q_{min}^{2/3}) / (E_{1max} – E_{1min})]^{3/2} \quad (9)$$

$$\Delta = E_{1max} -3/2 [(Q_{max} / B_0)^2 /g]^{1/3} \quad (10)$$

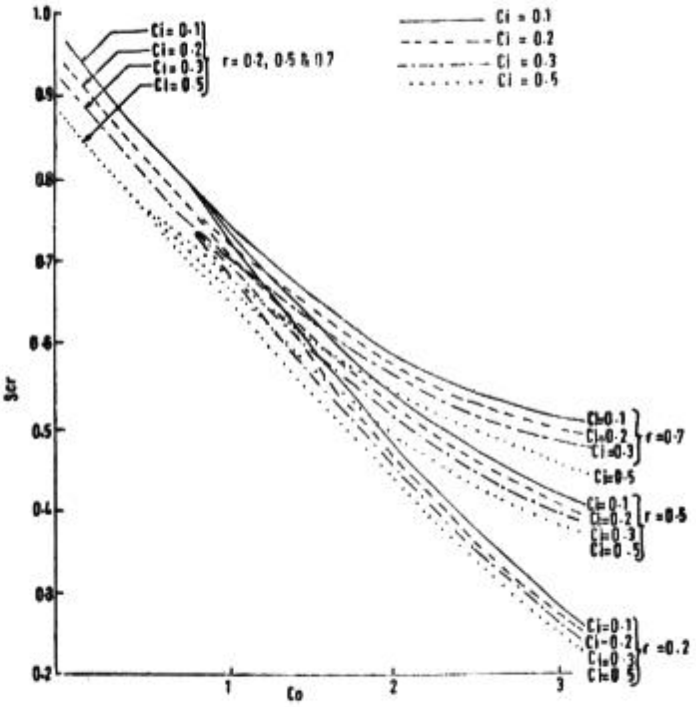

Fig. 3 Showing Variation of Modular limit (S_{cr}) with C_i ,C_o and r

Jaeger (1956) type inlet transition was provided to minimize head loss at entry (C_i) for smooth flow at the control section. Outlet loss coefficient (C_o) can be significantly reduced by preventing flow separation with adverse bed slope (β) corresponding to rate of flaring of side walls as illustrated in Fig.2. β-value can be found from equation (11) below (Mazumder-2017,2012,1994)

$$\beta = \tan\text{-}1[(2y_c/B_0)\{(\delta^2+\delta+1)/(2+\delta+\lambda+2\lambda\delta)\} \tan\theta \qquad (11)$$

where, $\delta=y_c/y_2$, $\lambda=B_1/B_0$ and θ =Angle of divergence of side walls downstream.

4.1 Experimental Investigation on Proportional Flow meter

Experiments were performed in the hydraulics laboratory of Delhi College of Engineering (Now Delhi Technology University) to find coefficient of discharge (C_d) and modular limit (S_{cr}) of the proportional flow meter shown in Fig.2. Jaeger type inlet transition having average side splay of 2:1 was adopted in all the experiments. Outlet expansion was straight and the length varied from 0:1(abrupt type as in case of classical bridges) to 3:1(Fig.2). In experiment nos. 1 to12 (table-1), downstream bed was kept level ($\beta=0^0$) and the rest of experiments were conducted with adversely sloping bed with β computed from equation 11 with θ-values corresponding to side splay 1:1, 2:1 and 3:1 as shown in Fig.2. Hydraulic efficiencies (η_i and η_o) were computed in all the cases from measured head losses at inlet (C_i) and outlet (C_o) from the relation

$$\eta_i=1/(1+C_i) \qquad (12)$$

$$\eta_o=(1-C_o) \qquad (13)$$

Table-1 summarizes the results obtained from the experiments. It may be noticed that C_d-values are almost the same as given by eq.7 which is based on the assumption C_i=0. The flow meter has high modular limit indicating that the flow is free up to a submergence varying from 90% to 95%. It may also be noticed that with level bed ($\beta=0^0$), separation of flow occurred downstream resulting in high degree of non–uniformity of flow downstream as the value of Corrioli's coefficient (α_2)- as computed from measured velocity distribution downstream- were very high. α_2 is given by the relation

$$\alpha_2=\sum (u^3 \, dA)/AV^3 \qquad (14)$$

where u is local velocity through an elementary area dA , A is the cross-sectional area of flow downstream and V is the mean velocity of flow downstream. If the flow downstream is uniform, i.e. u=V, α_2=1 (from equation (14). By providing adverse slope to bed (β) computed from equation (11), separation could be eliminated and high degree of uniformity of flow could be achieved as apparent from low α_2-values in table-1. The flow was found to be stable downstream. Unstable flow is found to attack stream banks causing erosion which needs costly protective works.

5.0 USE OF SMALL BRIDGES AND CULVERTS AS FLOW METER

Using the theory as discussed under section-4, small bridges and culverts can be conveniently designed as a proportional type flow meter for finding flow in the channel.

5.1 New Bridges

Waterway under the bridge/Culvert (B_o) and the corresponding bed elevation (Δ) should be decided by use of equation (9) & (10) respectively for the flow range Q_{max} and Q_{min}. While Q_{max} may be taken as design flood of 50 and 25 year return period for culvert and small bridges respectively.
Q_{min} can be found from known value of water level in the stream during lean flow. With above B_o and Δ-values, the bridge/culvert will act as a proportional flow meter and it can be used for stream gauging by simply measuring upstream water level, since discharge is independent of downstream water level. Jaeger or any other smooth transition connecting normal channel with constricted bridge opening may be used to minimize head loss (C_i). Equation (7) can be used for determining flow corresponding to any water level upstream. To avoid flow separation, straight expansive side walls with 2:1 and corresponding bed slope (β by Eq.11) may be provided. It is necessary to make the floor rigid with properly designed wing walls. To illustrate the design procedure, an example is worked out in annexure-I.

5.1 Existing Bridges/Culverts

All existing bridges and culverts can be used for flow metering by knowing C_d-values in eqw.(8) and the effective waterway under the bridge/culvert. It is, however, necessary to find whether the flow under the existing bridge/culvert is free or submerged. Submergence (S) may be found from the relation

$$S=y_2/y_1 \qquad (15)$$

where y_1 and y_2 are the depths of flow upstream and downstream of bridge respectively -both measured above the bed level at bridge section as shown in Fig.2

Modular limit or critical submergence (S_{cr}) for the given bridge/culvert, depending upon inlet and outlet loss coefficients (C_i and C_o) and the fluming ratio ($r=B_0/B_1$), should be found from Fig.3 (Mazumder and Joshi, 1981). Flow is free if S<Scr and submerged if S>scr. C_i and C_o –values for different entry and exit conditions are available in text books referred above. USBR (1968) recommends following values of C_i and Co for design of canal transitions (Table-2).

Table-1 Co-efficient of Discharge and Modular Limit of Proportional Flow Meter

Expt No. (1)	Q (LPS) (2)	Side Splay Expn. (3)	β° (4)	C_d ($m^{1/2}$/sec) (5)	Modular Limit (6)	% η_t (7)	% η_u (8)	α_2 (9)	Remarks (10)
1	20	0:1	0	1.67	0.90	86	28	6.00	Separation occurred on both sides. Large eddies on both sides of tail channel.
2	10	0:1	0	1.68	0.91	87	39	-	- Do -
3	5	0:1	0	1.72	0.91	88	43	-	- Do -
4	20	1:1	0	1.67	0.92	86	31	4.05	- Do -
5	10	1:1	0	1.69	0.92	86	40	-	- Do -
6	5	1:1	0	1.69	0.92	88	49	-	- Do -
7	20	2:1	0	1.70	0.96	87	31	3.26	Separation occurs on one side only. Large eddy on one side of tail channel
8	10	2:1	0	1.70	0.96	88	40	-	- Do -
9	5	2:1	0	1.73	0.96	89	50	-	- Do -
10	20	3:1	0	1.67	0.98	88	33	2.73	- Do -
11	10	3:1	0	1.69	0.98	90	42	-	- Do -
12	5	3:1	0	1.73	0.98	94	54	-	- Do -
13	20	1:1	16.6	1.67	0.92	87	90	1.30	Two symmetric small eddies confined within expansion reach. Uniform flow in tail channel
14	10	1:1	16.6	1.69	0.92	89	91	-	- Do -
15	5	1:1	16.6	1.69	0.92	93	99	-	- Do -
16	20	2:1	8.48	1.70	0.96	88	91	1.23	No separation, no eddy, flow in tail channel smooth and uniform
17	10	2:1	8.48	1.70	0.96	92	93	-	- Do -
18	5	2:1	8.48	1.73	0.96	98	99	-	- Do -
19	20	3:1	5.67	1.67	0.98	90	93	1.17	- Do -
20	10	3:1	5.67	1.69	0.98	95	94	-	- Do -
21	5	3:1	5.67	1.73	0.98	99	99	-	- Do -

Table-2 C_i and C_o – values for different types of inlet and outlet conditions

Type of transition	C_i	C_o
Warped type	0.10	0.20
Cylinder quadrant	0.15	0.25
Wedge type	0.20	0.30
Straight line type	0.30	0.50
Square ended type	0.30	0.75

IRC:SP:13(2004() defines critical submergence as

$$(y_1-y_2)/y_2=0.25$$

or

$$S_{cr} = (y_2/y_1)_{cr}= 0.8 \qquad (16)$$

Although IRC recommendation is not so scientific, still it may be used to find approximately whether the flow under the bridge/culvert is free or submerged by comparing actual submergence (S) with critical submergence (S_{cr}). In case the flow is free ($S<S_{cr}$), equation (7) may be used for computing flow. In case flow is submerged ($S>S_{cr}$), equation (8) should be used with reduced Cd-value depending upon degree of submergence. Coefficient of discharge under submerged flow condition is given in Parshall (1950), USBR (1968), Ackers et al (1978), Ranga Raju (1993) Mazumder (2007), King (1954), Bos (1975). Tyagi (1980) measured Cd- values under free flow (C_{df}) and submerged flow (C_{ds}) conditions and plotted the C_{ds}/C_{df} -values for different entry and exit conditions and plotted them against different degree of submergence (S). Obviously, C_{ds}/C_{df} =1 up to modular limit. At 100% submergence C_{ds}/C_{df}=0. Values of C_{ds}/C_{df} varied between 1.0 to 0.0 depending upon inlet and outlet conditions. Knowing the C_{df}-value and the degree of submergence, C_{ds} can be found from these plots. A typical plot of C_{ds}/C_{df} against submergence (S) as observed by Tyagi (1980) is shown in Fig.4. It may be seen that Cd-values variy widely under submerged condition. Even a small error in finding submergence (S) may cause large error in discharge.

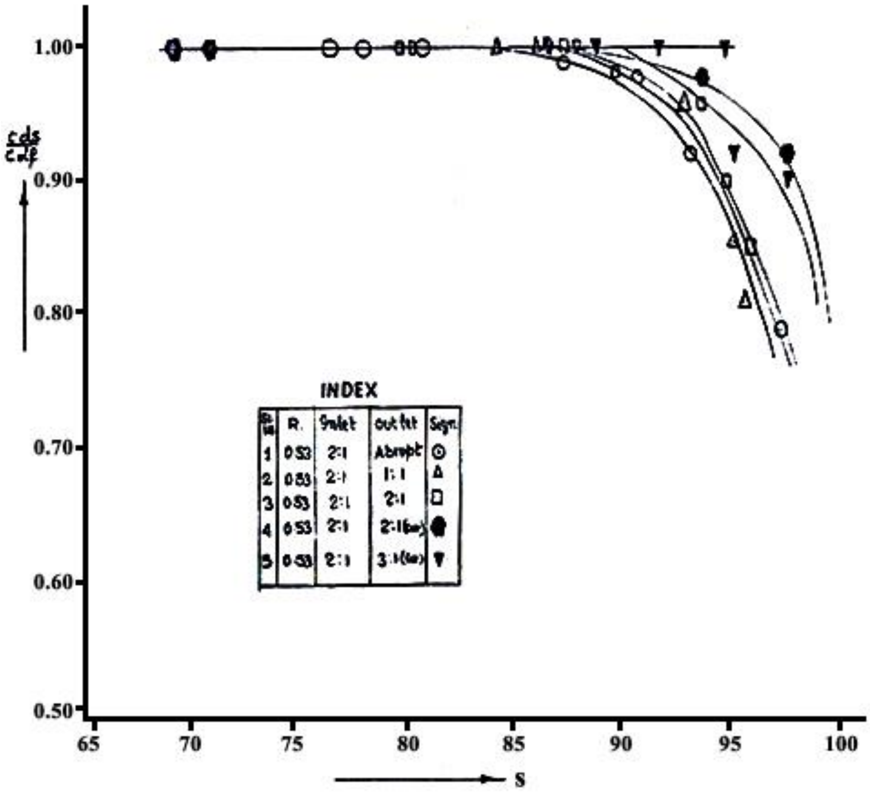

Fig. 4 Showing Variation of C_{ds}/C_{df} against Submergence(S)

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Annexure-1

A new bridge is to be constructed over a stream with data given below:

(i) Maximum design flood of 50 year return period – $Q_{max}= Q_{50}$=100cumec
(ii) Minimum flow in the stream- Q_{min}=20cumec
(iii) Flood level corresponding to Q_{max}=23.63m
(iv) Flood level corresponding to Q_{min}=21.38m
(v) Bed level at the bridge site- 20.00m
(vi) Mean width of stream at normal section=30m

Determine:

(i) the waterway (B_0) to be provided under the bridge and corresponding height of crest (Δ) above bed so that the bridge acts as proportional type flow meter under all approaching flow

(ii) Using the above geometries of the bridge, find flow in the channel when upstream water level is found to be 22.0m and justify the formula used for flow computation.

Solution: (i)

Corresponding to maximum flow: Q_{max}=100 cumec

y_{1max}=23.63-20.00=3.63m, V_{1max}=100/(30x3.63)=0.92m/s and

$E_{1max}= y_1+ V_1^2/2g=3.63+0.92^2/(2x9.8)=3.67$m

Corresponding to minimum flow : Q_{min}=20 cumec

y_{1min}=21.38-20.00=1.38m V_{1min}=20/(30x1.38)=0.48m/sec and

$E_{1min}= y_1+ V_1^2/2g=1.38+0.48^2/(2x9.8)=1.39$m

Using equation (9) and (10) :

$$B_0= [0.7 (Q_{max}^2 – Q_{min}^2) / (E_{1max} – E_{1min})]^{3/2}= [0.7(100^{2/3}-20^{2/3})/(3.67-1.39)]^{3/2}=9.07\text{m}$$

$$\Delta = E_{1max} -3/2 [(Q_{max} / B_0)^2 /g]^{1/3} =3.67- 1.5 [(100/9.07)^2/9.8]^{1/3}=3.67-3.47=0.20\text{m}$$

The bridge will act as a proportional flow meter, if the bed level under the bridge is raised by 0.2m i.e a bed level of 20.2m under the bridge. Provide upstream inlet transition (either Jaeger or any other smooth type) of axial length 2x1/2 (30-9.07)=20.93 say 20m. Since the flow will be free at all stages and C_d may be taken as 1.70 $m^{0.5}$/sec.(Table-1)

To avoid flow separation , non-uniform velocity distribution at out let and instability of flow downstream, it is recommended that a pair of side walls of axial length 20m be provided dowstream with advesely sloping floor having an angle β as follows:

$$\beta = \tan\text{-}1[(2y_c/B_0)\{(\delta^2+\delta+1)/(2+\delta+\lambda+2\lambda\delta)\} \tan\theta$$

where ,

y_c=2/3 E_{1max}=2/3x3.67=2.45m, y_c/B_0=2.45/9.07=0.27

$\delta = y_c/y_2 = 2.45/3.63 = 0.67$, $\lambda = B_1/B_0 = 30/9.07 = 3.30$, $\tan\theta = 1/2 = 0.5$

$\beta = \tan^{-1}[(2x0.27)\{ (0.67^2+0.67+1)/(2+0.67+3.30+2x3.30x0.67)\}]\ 0.5$

$= \tan^{-1}[0.055] = 3.5^0$

The depression of bed at the bridge exit will be=20xtanβ=1.10m

Bed level at exit of bridge =20-1.1=18.9m, The bridge and other appurtenant works is similar to Fig.2

Solution (ii)

Neglecting velocity of approaching flow in the first trial

$E_1 = 22.0 - 20.0 = 2.0m$

Assuming that the flow in the contracted section is at critical state under the bridge

$H = E_c = E_1 - \Delta = 2.0 - 0.2 = 1.8m$

Hence the discharge in the stream flowing under the bridge as per eq.(8)

$Q = 1.70\ xB_0xH^{3/2} = 1.70x9.07x(1.8)^{3/2} = 37.3$ cumec

To prove that the flow under the bridge is at critical state to justify that $C_d = 1.70$

$q_0 = Q/B_0 = 37.3/9.07 = 4.11$ cumec/m

$y_o = (q^2/g)^{1/3} = [(4.11)^2/9.8]1/3 = 1.2m$, when flow is critical

$V_0 = Q/B_0xy_0 = 37.3/(9.07x1.2)^{0.5} = 3.43m/s$

Hence Froude's number of flow under the bridge (F_0)

$F_0 = V_0/(gy_0)^{0.5} = 3.43/(9.8x1.2)^{0.5} = 1.0$

Since $F_o=1$ under the bridge, the flow is under critical state and it justified to use $Cd=1.70m^{1/2}/sec$