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Studies on Critical Submergence for Flowmeters

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1. Introduction

The present investigation is intended to study some basic principles governing submergence of flowmeters used for measurement of discharge in the field. A critical flowmeter is one in which critical flow occurs at the control section. For developing critical flow in a channel having subcritical slope, the channel may be constricted either laterally or vertically or both. So long, the flow in a flowmeter remains free or in other words the control section exists, discharge equation can be expressed as a function of upstream head only. There are situations when the tail water-level may be so high that a meter may be submerged. For submerged flow meters, the discharge is dependent on both the upstream and downstream depth.

A weir type critical flowmeter is shown in Figure 1. At a certain depth Y_2 downstream, hydraulic jump takes place (shown in full lines). Upstream depth is Y_1 at this instant. If downstream depth is increased beyond Y_2 (as shown by dotted lines), the front of hydraulic jump advances upstream, but there is no change in the upstream depth, Y_1. This happens upto when the downstream depth is equal to Y_2 (C_r) (chain dotted line). Beyond this depth Y_2 (C_r), any small increment in downstream depth $(\triangle Y_2)$ causes a corresponding increase in upstream depth $(\triangle Y_1)$. The flowmeter is said to be free for all downstream depth $Y_2 < Y_2$ (C_r) and it is submerged when Y_2 is $> Y_2$ (C_r). Critical submergence or modular limit is a partition between the free and submerged flow. Critical submergence ratio S_{cr} is defined as the ratio of the downstream depth to the upstream depth at the point of critical submergenced i.e., when $Y_2 = Y_2\,(C_r)$ and

$$S_{cr} = (Y_2/Y_1)C_r$$

Defining submergence as $S = Y_2/Y_1$, a flowmeter is said to be free when $S < S_{cr}$ and submerged when $S > S_{cr}$.

2. Historical Note

Studies were made by various authors in the past on the effect of submergence in the metering of an open channel flow.

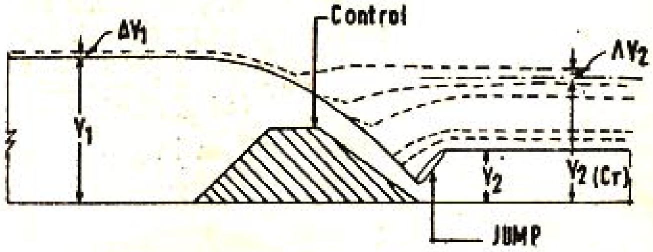

FIGURE 1 : Weir type critical flowmeter.

Parshall has developed a critical flow flume in 1920. Experimentally he had seen that submergence is to reduce the discharge.

USBR has defined critical submergence due to tail water for an ogee type spillway as

$$S = hd/H$$

Where, H is the head over crest and hd is the drop of tail water surface from upstream total energy line in a weir.

Scr as given by USBR is

$$S_{cr} = hd/H = 0.7$$

Balloffet developed an expression for the critical submergence for venturi type meters given by

$$S_{cr} = Y_2/Y_1 = 3\sqrt{2\mu^2}\left(\frac{3}{2} - .33787r^{2/3}\right)$$

$$\text{where } \mu = 2 \times \sqrt{\frac{1}{r^3}\,\text{Cos}^3\left(\frac{\pi}{3} + \frac{1}{3}\,\text{Cos}^{-1}r\right)}$$

r = fluming ratio = constant for a given meter

Y_2 is the downstream depth of water and S_{cr} is the critical submergence ratio.

Mazumder has developed following formulae for determining the critical submergence ratio for a pure venturi flume having horizontal bed.

$$\lambda_1^3 r^2 C_1 - \lambda_1 (2 + C_1) + 2 = 0$$

$$\lambda_2^3 r^2 C_2 - \lambda_2 (2 + C_2) + 2 = 0$$

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where, $C_1 = 1 + C_i$

$$C_2 = 1 - C_o$$

$\lambda_1 = Y_c / Y_1$, and $\lambda_2 = Y_c / Y_2$

Critical submergence is given by

$$S_{cr} = Y_2/Y_1 = \lambda_1/\lambda_2$$

He solved the above equations for different known inlet and outlet transitions, constriction ratio (r) and verified these results experimentally.

In the present study an attempt has been made to derive a set of general equations for any given flow meter having constriction both in the horizontal as well as vertical planes with different entry and exit transitions. Some of the analytical values were compared with actual critical submergence determined experimentally in order to verify the formula derived.

3. Analytical Determination of Critical Submergence

Figure 2(a) shows a critical type flowmeter having constriction in both horizontal and vertical plane, under free flow condition. Applying Bernoulli's equation between section 1-1 and 2-2

$$E_{f1} = E_{f2} + \Sigma H_L \quad .. \quad (1)$$

where, $\Sigma H_L = H_{Li} + H_{Lo} + H_{Lj}$

H_{Li} = Loss at inlet transition

$$= C_i \, (V_c^2/2g - V_1^2/2g) \quad .. \quad (2)$$

H_{Lo} = Loss at outlet transition

$$= C_o \, (V_c^2/2g - V_2^2/2g) \quad .. \quad (3)$$

H_{Lj} = Loss due to hydraulic jump.

V_1, V_c and V_2 are the mean velocities of flow at section 1-1, *c-c* and 2-2 respectively.

C_i and C_o are the coefficients of losses at entry and exit respectively.

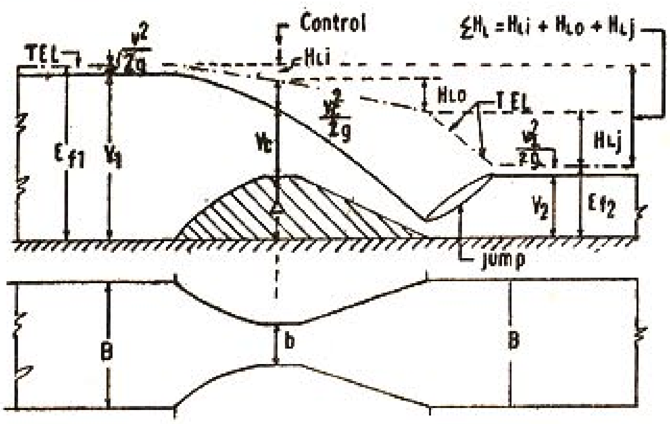

FIGURE 2(a) : Meter having free flow.

So long E_{f1} remains the same, Y_1 also remains constant. Therefore, the value of E_{f1} must remain constant upto critical submergence, beyond which the water surface upstream starts changing. When the downstream depth Y_2 is increased by a small amount $\triangle Y_2$, then E_{f2} also increase approximately by the same amount, since

$$E_{f2} = Y_2 + V_2^2/2g$$

As the velocity head $V_2^2/_{2g}$ is almost negligible compared to Y_2, with increase in E_{f2}, the value of ΣH_L must decrease by the same order of magnitude ($\triangle Y_2$) so that the sum of ($E_{f2} + \Sigma H_L$ remains the same as before. Since inlet and outlet losses remains constant the only way by which the total head loss (ΣH_L) can decrease is through reduction in head loss in hydraulic jump by the corresponding amount ($\triangle Y_2$).

It is possible to increase the downstream depth Y_2 without affecting Y_1 till the head loss due to hydraulic jump vanish. At this point of critical submergence as shown in Figure 2 (*b*)

$$E_{f1} = E_{f2} + H_{Li} + H_L \quad .. \quad (4)$$

Applying Bernoulli's equation between upstream (1-1) and critical section (c-c) at the limiting submergence condition [Figure 2(*b*)]

$$Y_1 + V_1^2/2g = (\triangle + Y_c) + V_c^2/2g + H_{Li} \quad .. \quad (5)$$

Putting the value of H_{Li} from equation (2) to equation (5)

$$Y_1 + V_1^2/2g = (\triangle + Y_c) + V_c^2/2g$$

$$+ C_i \left(\frac{V_c^2}{2g} - \frac{V_1^2}{2g} \right) \quad .. \quad (6)$$

From equation (6)

$$Ci \, \frac{Y_1 - (\triangle + Y_c)}{V_c^2/2g - V_1^2/2g} - 1 \quad .. \quad (7)$$

From equation of continuity

$$Y_1 \, V_1 \, B = Y_c \, b \, V_c$$

$$\text{or } V_1 = (Y_c/Y_1) \, r V_c \quad .. \quad (8)$$

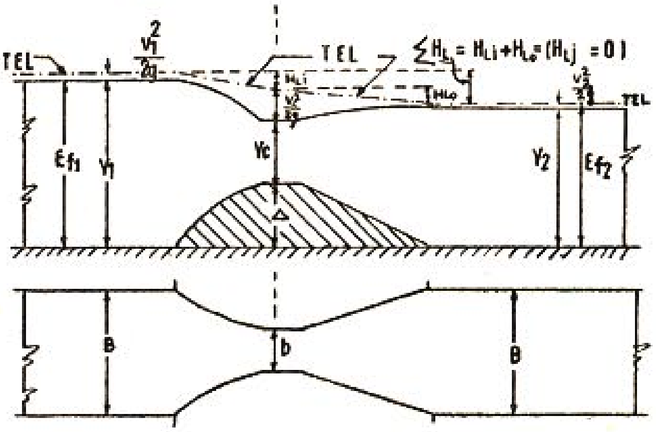

FIGURE 2(b) : Meter at critical submergence .

where $r = b/B$

From characteristic of critical flow

$$V_c^2/2g = Y_c/2 \quad .. \quad (9)$$

with above, equation (6) can be written as

$$Y_1 = \triangle + Y_c + (1 + C_i)V_c^2/2g[1 - (Y_c/Y_1)^2 r^2]$$
$$= \triangle + Y_c + (1 + C_i)Y_c/2\,[1 - (Y_c/Y_1)^2 r^2] \quad .. \quad (10)$$

Dividing both sides by Y_1 and substituting

$Y_c/Y_1 = \lambda_1$, $1 + C_i = C_1$ and $\triangle/Y_1 = R$

Equation (10) reduces to

$$\lambda_1 + R + C_1\lambda_1/2\,[1 - \lambda_1^2 r^2] = 1$$

$$\text{or } C_1 r^2 \lambda_1^3 - (2 + C_1)\lambda_1 + (2 - 2R) = 0 \quad .. \quad (11)$$

Similarly applying Bernoulli's equation between critical section (c-c) and the downward section (2-2) in Figure 2(b) at the limiting submergence condition, it may be proved that

$$C_2 r^2 \lambda_2^3 - (2 + C_2)\lambda_2 - 2R' + 2 = 0 \quad .. \quad (12)$$

where $\lambda_2 = Y_c/Y_2$, $C_2 = 1 - C_o$, $R' = \triangle/Y_2$

$$\& \qquad C_o = 1 - \frac{Y_2 - (\triangle + Y_c)}{V_c^2/2g - V_2^2/2g} \quad .. \quad (13)$$

Since $Scr = Y_2/Y_1 = \lambda_1/\lambda_2 = R/R'$

$\lambda_2 = \lambda_1/S_{cr}$ and $R' = R/S_{cr}$

Putting these values of λ_2 and R' in equation (12),

$$C_2 r^2 \left(\frac{\lambda_1}{S_{cr}}\right)^3 - (2 + C_2)\left(\frac{\lambda_1}{S_{cr}}\right) - 2\left(\frac{R}{S_{cr}}\right) + 2 = 0$$

On simplification, finally

$$2(S_{cr})^3 - (S_{cr})^2\,[(2 + C_2)\lambda_1 + 2R] + C_2 r^2 \lambda^3_1 = 0 \quad .. \quad (14)$$

Equation (14) will give the value of critical submergence for given boundary conditions after λ_1 is found out by solving Equation (11).

Since it is rather laborious and time consuming to solve values of λ_1 and Scr for every individual case, computer programming was made for solution of the above equations. Computer programming is given in Annexure-A. IBM-360 computer at Delhi University was used for analytical determination of S_{cr} with Equations (11) and (14) for the different values of parameters given below

$r = 0.2, 0.5, 0.7$

$R = 0.2, 0.4, 0.6, 0.9$

$C_i = 0.1, 0.2, 0.3, 0.5$

$C_o = 0.1, 0.4, 0.7, 1.0, 2.0, 3.0$

288 values of S_{cr} as determined above are plotted in Figures (3) to (6).

4. Experimental Set-up

Experiments were carried out in the hydraulics laboratory of Delhi College of Engineering Delhi. The experimental set-up was installed in a tilting, self circulating flume 3 meter long and 30 cm in width. Wooden models were prepared and twenty experiments were conducted using six different models individually or in combination for the different inlet and outlet transitions. Some typical cases are drawn in Figure 7(a) to (d). Transition in the vertical plane were made with cement mortar. In all the experiments r and R was kept constant, namely, 0.48 and 0.45 respectively.

Discharge was kept constant at (7 lit./sec.) radial gate in the downstream end was operated to control the water surface reading at downstream and upstream. Upto a certain depth of Y_2, Y_1 remained constant. After the critical point, any rise in tail water (Y_2) caused simultaneous rise in Y_1. The meter was submerged. Few readings of Y_1 and Y_2 were taken before and after submergence. The values of Y_2 and Y_1 so recorded were plotted as shown in some typical Figures 7(a) to (d). The critical value of Y_2 was obtained from the above figure and the critical submergence was determined experimentally as shown in Figures 7(a) to (d).

The critical depth Y_c is determined from the formula

$$Y_c = (q^2/g)^{1/3}$$

where $q = Q/b$

The values of S_{cr} so determined for the various entry and exit conditions are given in Table I.

5. Analysis of Results

The analytical values of S_{cr} for various values of Ci, C_o, r and R have been plotted graphically with C_o in the abcissa and S_{cr} in the ordinate in Figures 3 to 6. For all the figures plotted, it may be found that S_{cr} decreases with increasing values of Ci and C_o. Comparing Figures 3 to 6, it can be stated that the range of variation in S_{cr} with Ci and Co decreases gradually as the value of R goes on increasing. It may be concluded, therefore, that for high values of R, S_{cr} is not much affected by the inlet and outlet transitions (Ci and C_o) as compared to low values of R.

From Figures 3 to 6 it may be seen, as r increases, S_{cr} also increases. The effect of r on S_{cr} is however, dominant only for high values of Co, (Figures 3 and 4). At low values of Co, r has practically no effect on S_{cr}. Even for high values of C_o, the range in variation of S_{cr} with r decreases as R increases. For high values of R, effect of

r on S_{cr} is practically insignificant for all values of C_o. It may further be concluded that S_{cr} increases as R increases. At high values of R, the effect of Ci, C_o and r on S_{cr} is practically insignificant compared to those at low values.

Values of S_{cr} determined experimentally corresponding to the various inlet and outlet conditions are tabulated in column 7 of Table I. Coefficients of head loss C_i and C_o have been computed for all the experiments by Equations (7) and (13) and are tabulated in columns (4) and (6). Values of r and R were kept constant at $r = 0.48$ and $R = 0.45$ Corresponding to given values of R, r, Ci and C analytical values of S_{cr} given by equations (11) and (14) is found by interpolation of Figures 4 and 5 and are tabulated in column (6). Comparing the analytical and experimental values of S_{cr}, it is found that the maximum error is about 1.5 per cent in experiment No. 5 and the minimum error is 0.11 per cent in experiment No. 19. It may be concluded, therefore, that the values of S_{cr} given by Equations (11) and (14) are correct and the above equation may be used for determination of critical submergence almost precisely.

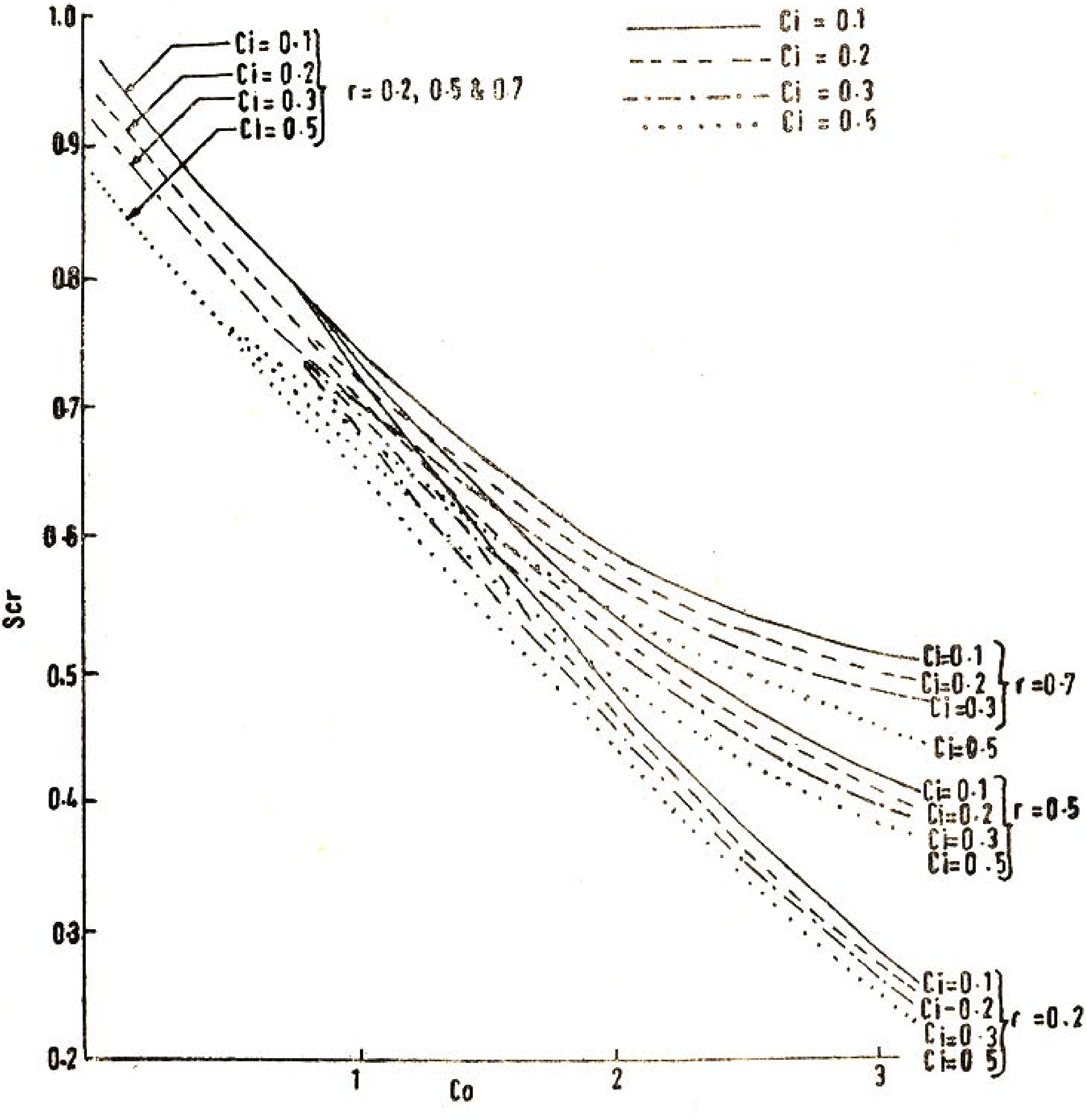

FIGURE 3 : Variation of critical submergence with C_i, C_o and r.

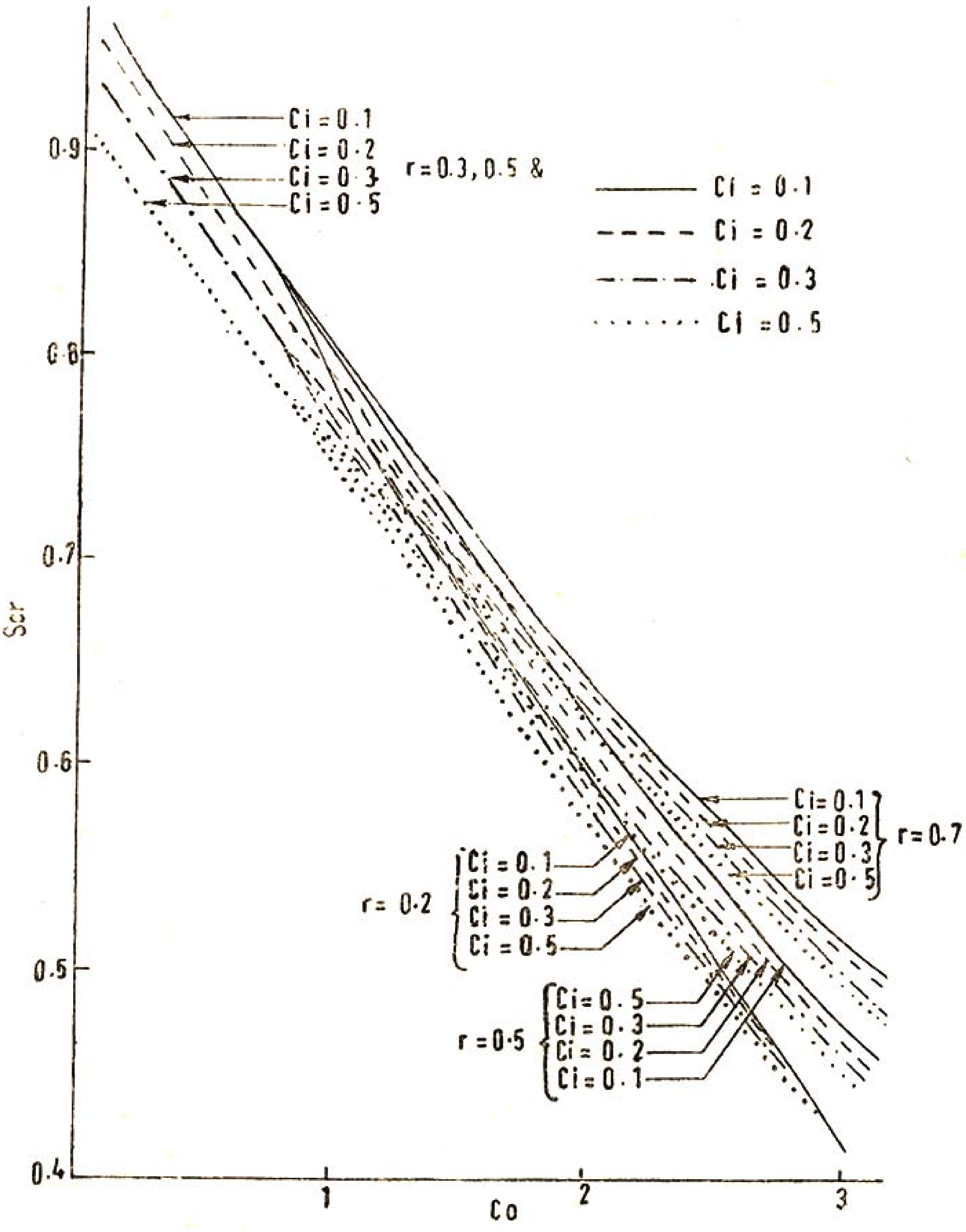

FIGURE 4 : Variation in critical submergence with C, C and r.

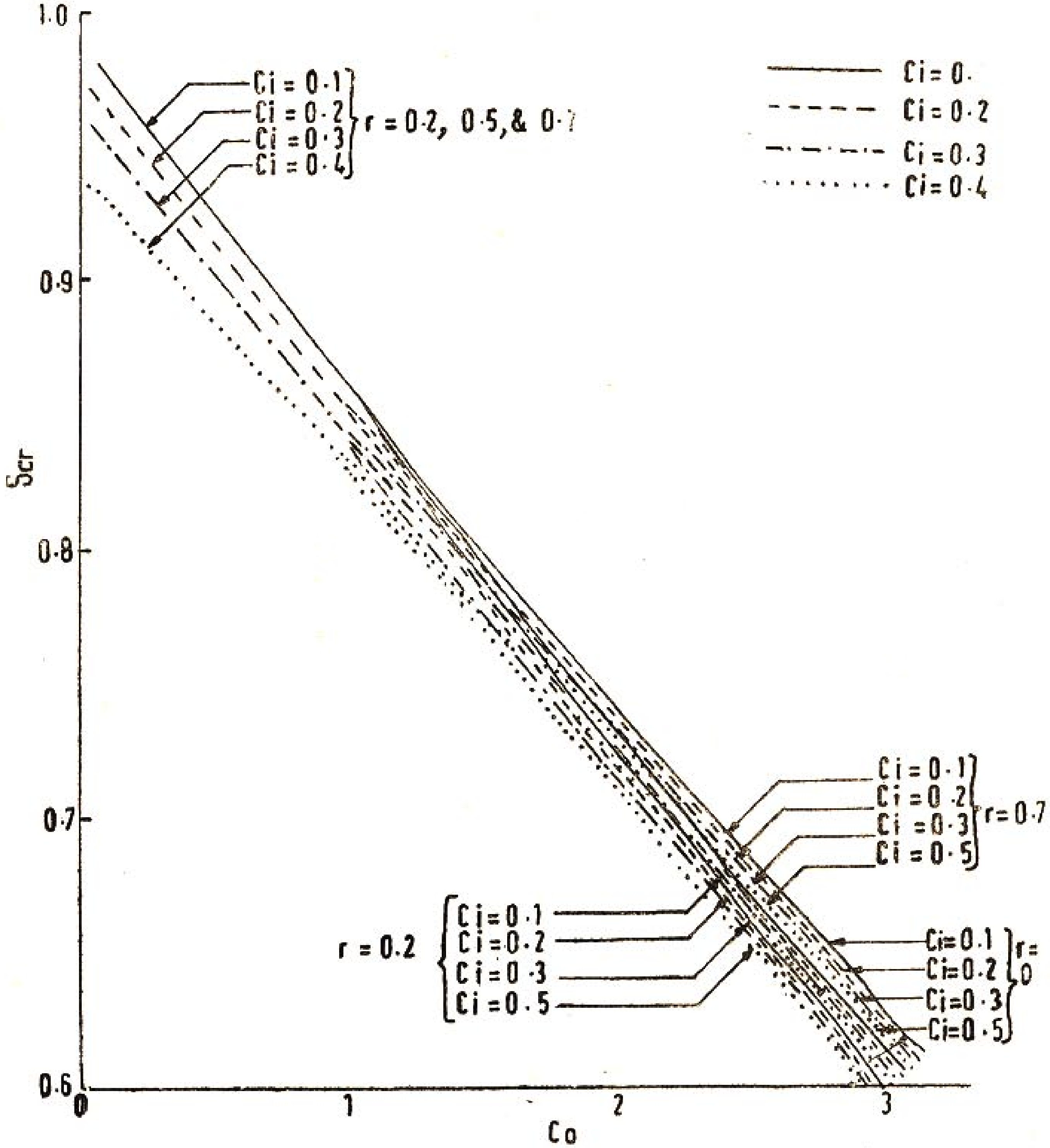

FIGURE 5 : Variation in critical submergence with Ci, Co and r.

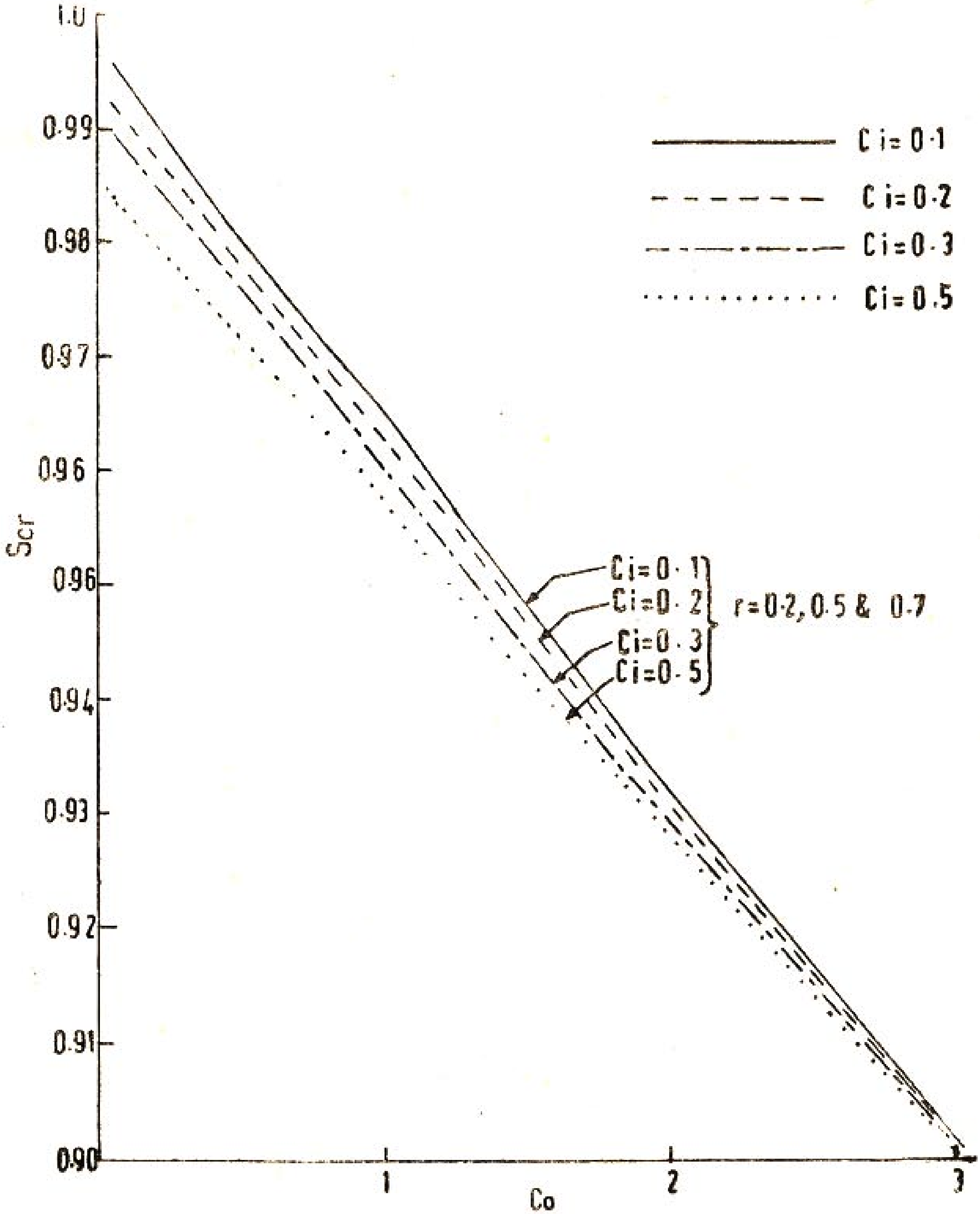

FIGURE 6 : Variation in critical submergence with Ci, Co and r. R=0.9

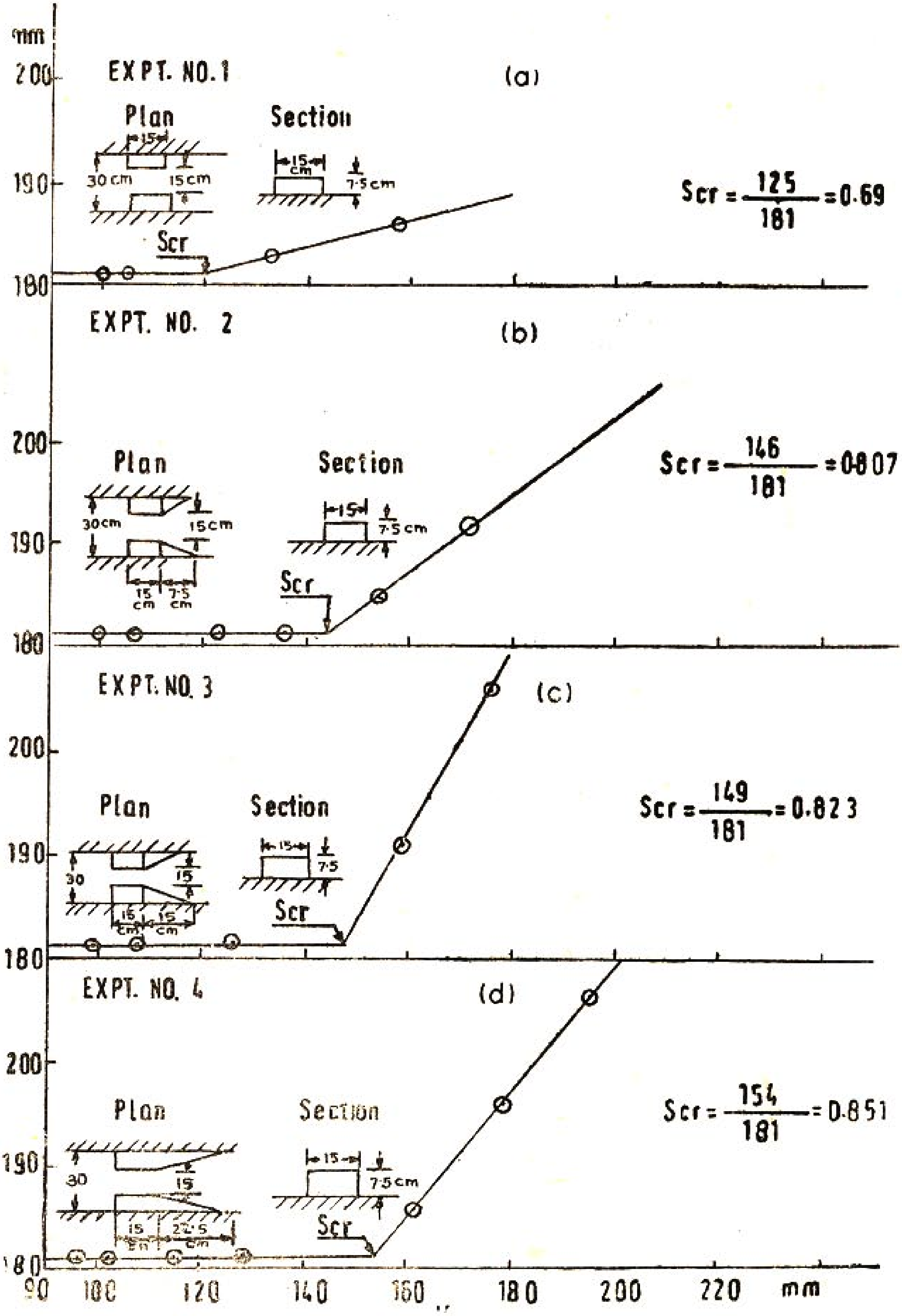

FIGURE 7 : Typical experimental values of Scr.

TABLE I

Comparison between theoretical and experimental values of critical submergence ratio

Column No. (1) Sl. No. of Experiment	(2) r	(3) R	(4) C_i	(5) C_o	(6) S_{cr} (Analytical)	(7) S_{cr} (Experimental)
1	.48	.45	.56	1.37	0.70	0.69
2	.48	.45	.56	0.65	0.81	0.80
3	.48	.45	.56	0.53	0.83	0.82
4	.48	.45	.56	0.36	0.86	0.85
5	.48	.45	.56	0.26	0.88	0.86
6	.48	.45	.51	1.18	0.73	0.72
7	.48	.45	.51	0.54	0.84	0.82
8	.48	.45	.51	0.37	0.86	0.85
9	.48	.45	.51	0.26	0.89	0.88
10	.48	.45	.51	0.16	0.89	0.89
11	.48	.45	.50	1.09	0.75	0.74
12	.48	.45	.50	0.40	0.86	0.85
13	.48	.45	.50	0.30	0.88	0.87
14	.48	.45	.50	0.13	0.90	0.89
15	.48	.45	.50	0.09	0.90	0.90
16	.48	.45	.42	0.57	0.84	0.83
17	.48	.45	.42	0.40	0.97	0.86
18	.48	.45	.42	0.30	0.88	0.88
19	.48	.45	.42	0.13	0.91	0.91
20	.48	.45	.42	0.09	0.91	0.91

6. Conclusions

(1) Critical submergence (S_{cr}) of a flow meter is primarily dependent on (*i*) Inlet loss coefficient (C_i), (*ii*) Outlet loss coefficient (C_o), (*iii*) Horizontal constriction ratio (r) and (*iv*) Vertical constriction ratio (R).

(2) S_{cr} increases as R and r increases.

(3) S_{cr} increases as C_i and C_o decreases.

(4) At high value of R, the relative effect of C_i, C_o and r on S_{cr} are not significant, when compared to those at low values of R.

(5) At every low values of C_o, effect of r on S_{cr} is insignificant.

(6) Equations (11) and (14) may be used for precise determination of critical submergence of any flow meter. When $r=1$, the equations give S_{cr} for weir extending full width of the channel and with $R=0$ the equations will give S_{cr} for pure venturi type flume having level bed.

(7) The conventional method of defining critical submergence for a weir when the tail water-level just touches the crest level is not correct as the critical submergence really depends on the type of entry and exit transitions and also on the amount of horizontal and vertical constriction. Thus a meter may remain free when the tail water is above the crest level and it may be submerged even when tail water is below the crest level.

7. Acknowledgement

The present investigation forms a part of Thesis work done by the second author under the guidance of the first author. Authors express their gratefulness to Delhi College of Engineering Authorities for providing necessary facilities to conduct experiments and to the University of Delhi for allowing free use of computer. They also thank all the staff members who have helped during experiments and computational works.

8. References

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ANNEXURE—A

```
IA 551    003245
//LMJO   JCB      DCE/79. CE.50
*M : 781
//DCE CDT EXEC FORTRAN (BCC, MAP)
              DIMENSION R1(3), R(4), C(4), C2 (6)
                C1 (100), C0 (300)
              READ 20, (R1 (I), I=1, 3)
              READ 20, (R (J), J=1, 4)
              READ 20, (C (K), K=1, 4)
              READ 20, (C2 (L), L=1, 6)
        20    FORMAT (17F4.1)
              DO  60 I=1, 3
              DO  60 J=1, 4
              DO  60 K=1, 4
FORTRAN IV MODEL 44 MFT VERSION 3, LEVEL 4
DATE 79/041
              M=0
              X=1.0
        10    FX=(C(K)*R1 (I)*R) (I) (X)**3-
                (2+C(K))*X-2*R(J)+2
              FFX=3*C (K)*R1(I)*R1 (I)*X*X-2-C
                (K)
              F=-FX/FFX
              M=M+1
              X=X+F
              IF (M-10) 10, 25, 25
        25    DO  60 L=1, 6
              Y=1.0
              M=0
        30    FY  2*Y**3-Y*Y* (X*(2+C2(L))
                +2*R(J)*C (L)*R1 (I *R1(I)*X**3
              FFY=3*Y*Y-2*Y*(X*(2+C2(L))
                +2*R(J)
              FY=-FY/FFY
              M=M+1
              Y=Y+FY
              C1(K)=C(K)-1
              CO(L)=1-C2(L)
              IF(M--10) 30, 60, 60
        60    PRINT 45, R1 (I), R(J), C1(K), CO(L), Y
        45    FORMAT (4F20.1, E 30.8)
              STOP
              END
```