

CONTROL OF SHOCK WAVES IN SUPERCRITICAL EXPANSION

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INTRODUCTION

Supercritical expansions are provided in hydraulic structures, e.g., Chutes, outlets, spillways, etc. Most often it is avoided due to disturbance created by shock waves downstream of the expansion. There is possibility of reducing the cost of the structure by narrowing the chute and introducing expansion at the bottom of chute. Unlike subcritical flow, supercritical expansion is subjected to undulation of water surface due to reflections of shock waves from the side walls. Higher is the Froude's number of the incoming flow, larger will be the disturbance and longer is the stretch of tail channel affected by shock waves. Height of the side-walls has to be raised to contain the flow level which increases at all the reflecting points. Rouse's expansion is very effective in controlling shock waves. But it is costly for its long length and higher walls. It has another limitation where hydraulic jump is likely to occur. In case the jump front moves inside the expansion, flow separates from the boundary resulting in jet flow along one of the side walls and violent eddies along the opposite side. This is likely to cause damages to the channel and the structure. Rouse (1951) proposed a sudden drop at the exit to stabilise the jump. Mazumdar (1987, 1988) used bed deflector and adverse slope to the expansion floor to stabilize jump within the expansion. Molino (1989) used hump, bottom traverse and teeth to control expansion flow. Vischer (1988) recommended domed bottom for an expanding chute. Various devices used for controlling shock waves in different types of expansions are discussed below.

MECHANISM OF SHOCK WAVES

When an incoming flow boundary AB (Figure 1) is deflected through an angle θ (Point B), disturbance propagates along a line inclined at an angle β . The

incoming stream maintains the initial direction upto the line of disturbance, known as shockwave (also shock front), which deflects the stream through angle θ making it parallel to the new boundary BC. Similar disturbance is created at point C where the boundary CD is again deflected. Upper portion of the boundary ABCD represents flow contraction while the flow in the lower side resembles expansion.

The first shock front BE in the contraction is called positive shock, since the boundary moves towards the flow. Shock front BF in the expansion, which is a mirror image of BE (with respect of AB) is a negative shockline as the boundary moves away from the flow. While the flow depth increases all along the positive shock front, there is reduction in flow depth along the negative shock front. For the same reasons, the shock fronts CG and CH are positive and negative shocks respectively. Similar shock waves B'E and C'G are produced from the other side wall [Figure 1(b)]. The positive shock fronts cross each other at points G, I, J, etc., at the centre line and are reflected from the side walls at 2-2', 4-4', 6-6', etc., with rise in water surface. At all such points 1-1', 3-3', 5-5', etc., where the negative shocks get reflected, there is fall of water surface. Thus the flow surface along the wall (as well as other longitudinal sections) will be undulating with consecutive rises and falls. Shock waves are in essence the natural process by which the flow regains its original state of smooth and uniform nature. Figures 2(a), (b), (c) illustrate typical shock waves and flow pattern downstream of expansions with different shapes of boundary.

DIFFERENT PRINCIPLES OF CONTROLLING SHOCK WAVES

Shock waves may be controlled by any one or in combination of the methods stated below:

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(a) **By hydraulic jump:** Classical shock waves are due to the reflection of the waves oblique to the walls. Hydraulic jump is a shock wave where the jump front (or shock front) is at right angle to the walls. If the expansion ends in a hydraulic jump at the exit end, there is no shock wave downstream where the flow is at sub-critical stage. It is, however, associated with sudden rise in tail water level and dissipation of energy.

(b) **By superimposition of positive and negative shock fronts:** Oblique shock waves can be controlled through superimposition of positive and negative shocks. Ippen (1951) utilized this principle for the design of supercritical contraction. Rouse's (1951) supercritical expansion is also based on the principle of superimposition.

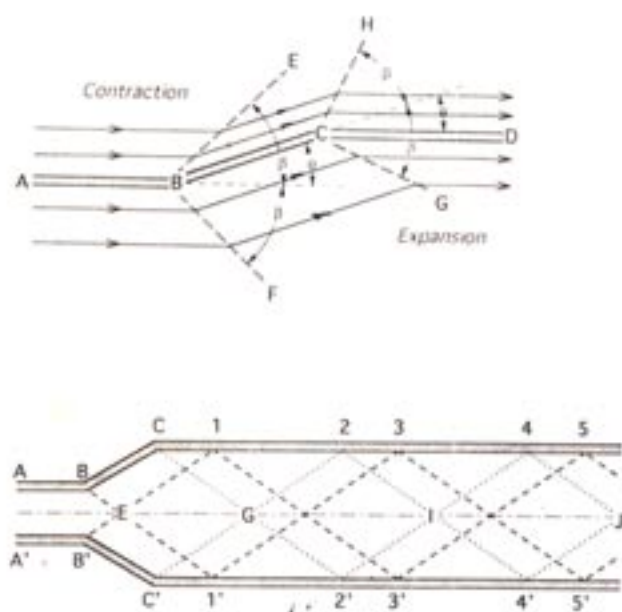


Figure 1 (a) Mechanism of Shockwave Generation
(b) Reflection of Positive (.....) and Negative (----) Shockwaves from side walls

(c) **Interaction between two positive shock waves:** In nature, positive shock waves from opposite walls intersect with each other giving rise to ripples which are gradually washed out through friction. Nearer the expansion exit this interaction begins, earlier the shock waves disappear. It may be possible to generate artificial positive shock fronts close to the exit end of expansion so as to restore the normal state of flow nearer to the exit of expansion.

(d) **Choking of flow:** Shock waves are formed only when the flow is supercritical. With choking of flow, Froude's number is unity at the control section resulting in elimination of shocks. Various ways of choking of flow in a supercritical expansion are discussed below:

(i) **Raising the floor level:** With reference to Figure 3, an incoming supercritical flow q_1 (Point 1), is required to be changed to another supercritical flow q_2 (Point 2), where q_1 and q_2 are

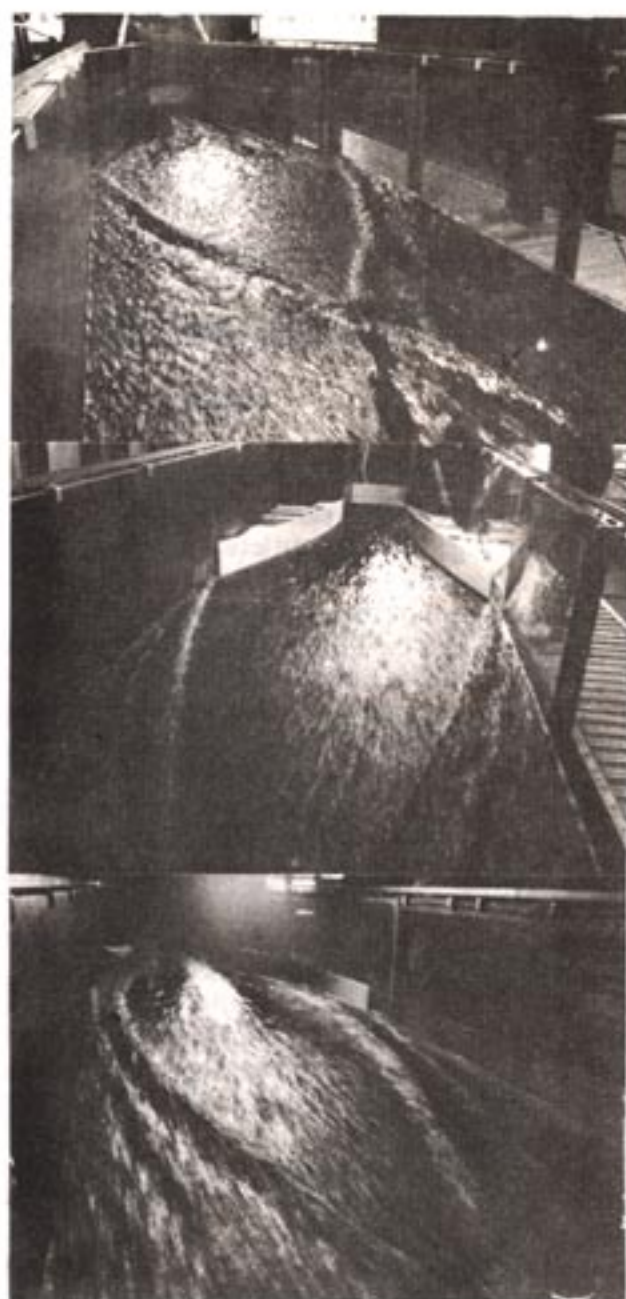


Figure 2: Shockwaves in Expansions, $h_0 = 48$ mm (a) Straight boundary, $F_0 = 4$ (b) Rouse Modified Boundary (Rouse M1), $F_0 = 1$, $F_1 = 8$ (c) Rouse Reverse Boundary (Rouse R1) $F_0 = 1$, $F_1 = 4$.

the intensities of flow at the entry and exit ends of expansion respectively and are given by $q_1 = Q/B_1$ and $q_2 = Q/B_2$ where Q is the total flow, B_1 and B_2 are the widths of expansion at entry and exit respectively. Instead of following the route [1-2 (Figure 3)] the flow expansion is possible also along the route 1-C₂-2 by raising the floor level at exit by an amount ΔZ_2 so that the flow is at critical stage with control at the exit of the expansion. Another way of choking the flow is to follow the route 1-C₁-2 by raising the floor level at entry by ΔZ_1 so that the flow is at critical stage with control at the entrance to the expansion. Obviously, the first method is better than the second, since flow downstream of expansion will be shock free irrespective of the expansion boundary. However, $\Delta Z_2 > \Delta Z_1$.

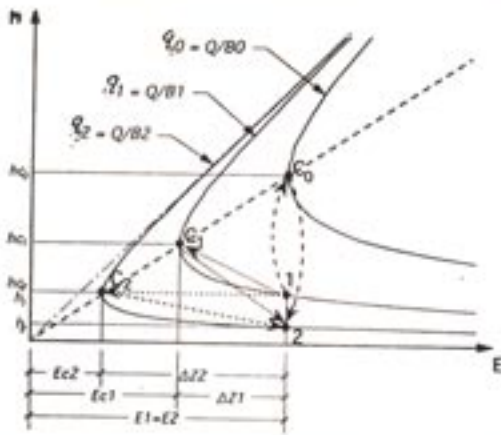


Figure 3: Specific Energy Diagram indicating different methods of flow choking

(ii) *Contraction of flow before entry:* If the initial flow width B_1 is contracted to width B_0 , incoming flow intensity q_1 increases to $q_0 (=Q/B_0)$. The flow can be choked by choosing B_0 (or q_0) such that the route 1-C₀-2 (Figure 3) is followed. Such contractions can be designed so that the positive shock fronts produced at contraction meet the expansion walls tangentially and cancel the negative shock waves, resulting in smooth flow free from shock waves.

(iii) *Simultaneous contraction and raising floor level:* Flow can be choked by suitable

combinations of methods (i) and (ii) so as to ensure critical flow under all the flow conditions.

EXPERIMENTAL SET UP

All the experiments were performed in the Laboratory of Hydraulic Constructions at EPFL, Switzerland. Experimental set up is shown in Figure 4. By controlling the flow (upto 250 L/s³) and the gate opening, Froude's number of incoming flow F_0 (of width $b_0 = 50$ cm) could be varied from 1 to 10. Tests were made with two different inflow depths (h_0) of 48 mm and 96 mm corresponding to aspect ratios (b_0/h_0) 10.4 and 5.2 respectively. Discharge was controlled by a calibrated electronic control panel. Depths of flow were measured correct upto 0.5 mm by means of an electronic device of air gap sensitive pointer gauge (Jayaraman, 1973) which ensured exact point of contact with water surface. Velocities were measured with a micropropeller connected to an electronic recorder. Angles of the velocity vectors were measured with an angle meter provided with a highly sensitive vane at the bottom which orients along the flow, indicating exact angle on a circular disc at the top. Co-ordinates of shock waves were measured with pointer gauge which could be moved at any point of the flow surface by moving the gauge over a trolley which rested on rails. Computations were made with a Macintosh Computer using Microsoft Excel worksheet.

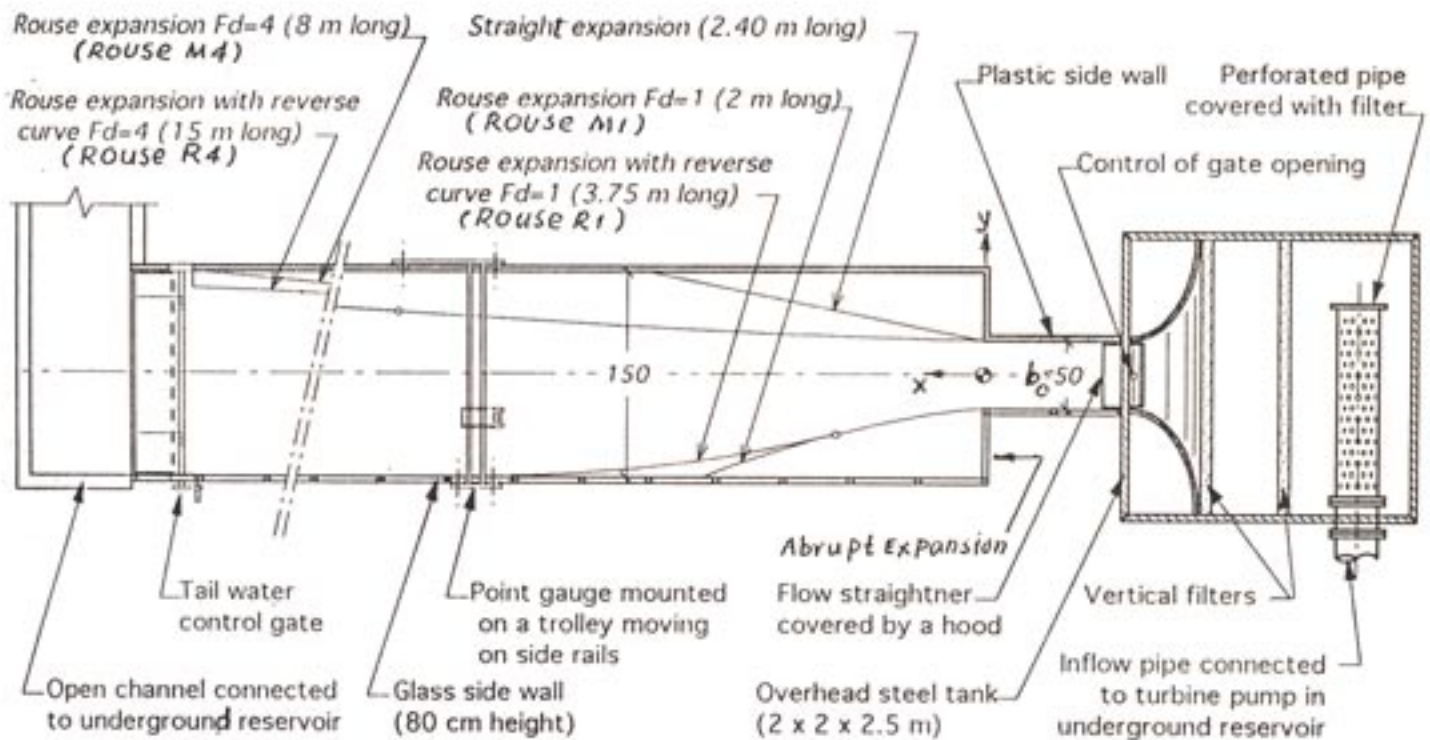


Figure 4: Experimental Set up showing different Expansion Boundaries

- Note: (i) Rouse Modified Expansion : $y/b_0 = 1/2 [1 + 1/4 X^{3/2}]$, where $X = x/b_0 F_0$, $F_0 = 1$ for Rouse M₁ & $F_0 = 4$ for Rouse M₄
 (ii) Rouse Reverse Expansion : See reference - Rouse (1951) $F_0 = 1$, for Rouse R₁, $F_0 = 4$ for Rouse R₄.

PERFORMANCE CRITERIA

Results of the experiments with and without shock control devices have been compared in terms of some performance criteria described below:

(i) *Waviness Factor (W_f)*: It gives a measure of waviness of the flow surface and is expressed as $W_f = (S/b-1)$ where S is the length of water surface (across the channel of width b) which was magnified by plotting $(h_i/h)^2$ values at several points ($i=1, 2, 3, \dots, n$) in the section. W_f is zero in case of a perfectly smooth surface where $s=b$.

(ii) *Coriol's Coefficient (α)*: It is a measure of non-uniformity in velocity across any section and is given by the expression

$$\alpha = 1/AV^3 \int_A u^3 dA \quad \dots(1)$$

where u is the velocity of flow through an elementary area dA in any flow section of area A . V is the mean velocity of flow across this section. When $u = V$, $\alpha = 1$, which indicates perfectly uniform distribution of velocity in the given section. Higher the non-uniformity of flow greater is the α -value.

(iii) *Lateral Momentum Transport Factor (T_f)*: It is given by the ratio of lateral momentum (M_y) to axial momentum (M_x) across a control section normal to the velocity vectors.

$$T_f = \frac{\sum M_y}{\sum M_x} \quad \dots(2)$$

when the flow is axial, $M_y = 0$ and hence $T_f = 0$.

(iv) *Relative loss in energy ($\Delta E/E_1$)*: ΔE is the loss of energy between entrance and any section downstream and is given by $\Delta E = E_1 - E_x$ where E_1 is the energy head at entrance and E_x is the energy head at a distance x downstream from the entrance. The energy head

$$E_x = h_{mx} + \alpha_x \cdot V_x^2 / 2g \quad \dots(3)$$

where α_x is the Coriol's coefficient at any section at a distance x from the entrance and V_x is the mean velocity of flow across the section. Since the depths

vary substantially across the section in supercritical flow with shock waves, h_{mx} was computed from the relation

$$h_{mx} = 1/Q \sum_{i=1}^{i=n} u_{xi} \cdot h_{xi}^2 \cdot dy \quad \dots(4)$$

where u_{xi} is the axial component of velocity over an elementary width dy and h_{xi} is the depth of flow at several points ($i=1,2,3,\dots,n$) in the section at a distance x from the entrance. In case of uniform flow ($h_x = h_u$) with uniform velocity distribution ($u_x = V_x$), $h_{mx} = h_x$.

The values of the above parameters are given in Tables 1 and 2.

RESULTS

Results obtained without and with shock control devices have been compared in Tables 1 and 2 in terms of the various performance criteria as already discussed. All the values refer to a given section at a distance of 7m ($x=7m$) measured from entry to the expansion. The inflow Froude's number (F_o) was kept same in all the cases, i.e., $F_o=4$. Aspect ratios were 10.4 ($h_o=48$) and 5.2 ($h_o=96$). Results are summarized separately for the different types of expansions as follows:

(a) **Abrupt Expansion**: Performance of abrupt expansion is given in Table 1. No attempt was made to control shock waves. For further details refer Hager and Mazumder (1992).

(b) **Straight Expansion**: As shown in Figure 4, a straight expansion having 2.4 m axial length (4.8:1 side splay), was first tested with level bed. Results are given in Table 1. Shock wave is shown in Figure 2(a). Performances of the straight expansion with three different adverse bedslopes ($\theta = 1.7^\circ, 3.4^\circ$ and 5°) is given in Table 2. Comparing the corresponding values of the parameters $W_f, \alpha, T_f, \Delta E/E_1$ in Tables 1 and 2, it may be concluded that adverse slope is effective in controlling shock waves and improvement of performance. Figures 5(a) and (b) illustrate the flow condition with 5° adverse slope at $F_o=8$ & 4 respectively. Flow is virtually free from shock waves after the meeting of two weak fronts (Figure 5(a)). Flow was choked at $F_o=4$ and $h_o=48$ mm resulting in complete elimination of shock waves downstream (Figure 5(b)).

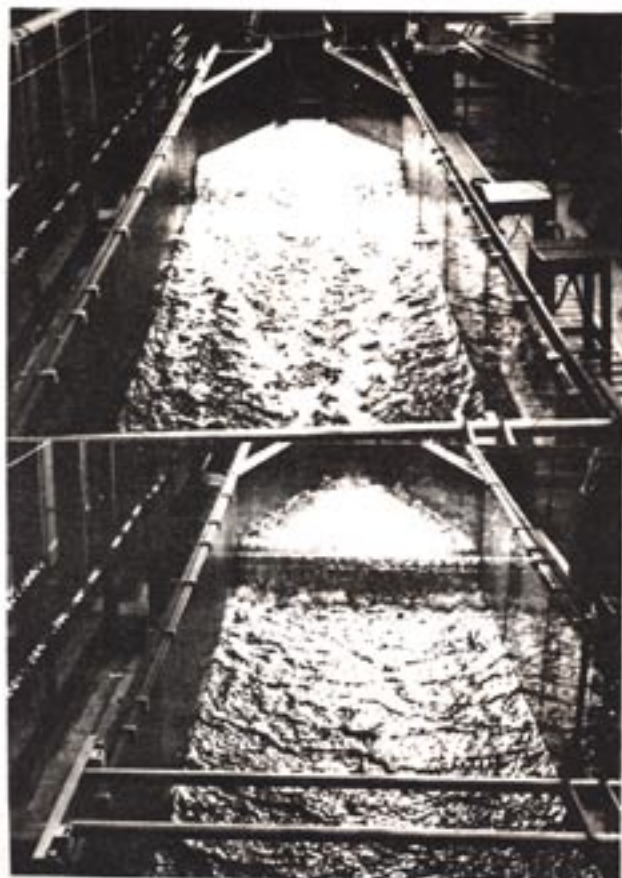


Figure 5: Shockwave Control in Straight Expansion with Adverse Bed Slope, $h_0 = 48$ mm, $\alpha_d = 5^\circ$. (a) $F_o = 8$ (b) Choked flow, $F_o = 4$.

(c) **Rouse Modified Expansion**: Rouse's modified equation, with design Froude's No. $F_D = 1$, (Rouse M_1) gives 2m and (Rouse M_4) for $F_D = 4$ gives 8 m long negatively curved expansion as shown in Figure 4. This type of expansion boundary is usually adopted in outlet works for efficient flow expansion with hydraulic jump at the exit. In supercritical expansion (without jump), however, it gives large shock waves (Figure 2b) due to high deflection of the flow at exit. The performance of the expansion without shock control device is given in Table 1, under Rouse M_1 and Rouse M_4 . For further details refer Mazumder and Hager (1993). Devices used for controlling shock waves are as follows:

(i) **Abrupt flow deflector**: Figure 6(a) shows the abrupt flow deflector. In this method, the Rouse's modified expansion (Rouse M_1) which is highly efficient in uniform spreading of flow was expanded to a width (B_2) slightly more than the final width B_2 and then contracted to width B_2 . Artificial shock fronts, generated by the abrupt contraction on either side, deflected the jet fanning out of the expansion and made it axial. This helped in controlling shock waves appreciably. The performance and effectiveness of the deflector were found to be governed by its location (x_i/L_1), contraction ratio $\gamma = (1 - B_2/B_2)$ and the angle of the deflector (α_d) as shown in Figure 6(a). For $F_o = 4$ and $h_0 = 48$ mm, values of the governing parameters were found to be:

$$\begin{aligned} x_i/L_1 &= 0.25 \\ r &= (1 - B_2/B_2) = 0.16 \\ \alpha_d &= 75^\circ \end{aligned}$$

Performances obtained with abrupt deflector are given in Table 2. When the deflector was brought nearer ($x_i/L_1 < 0.25$), shock fronts moved inside the expansion making the deflector ineffective. The deflector was found to be ineffective at $F_o = 6$ and 8 also.

(ii) **Cusp type flow deflector**: Efficiency of the flow deflector increased remarkably by providing cusp type geometry as shown in Figure 6(b). Performances of the deflector with $x_i/L_1 = 0.35$, $r = 0.25$ and $\alpha_d = 45^\circ$ are given in Table 2 and the flow patterns are shown in Figure 7(a) for $F_o = 6$, Figures 7(b) and (c) for $F_o = 8$. Shock waves however reappeared at $F_o = 2, 3$ and 4, perhaps due to greater contraction. It may be possible to make the cusp-type deflector effective over a wider range of F_o by suitable combination of x_i/L_1 , r and α_d .

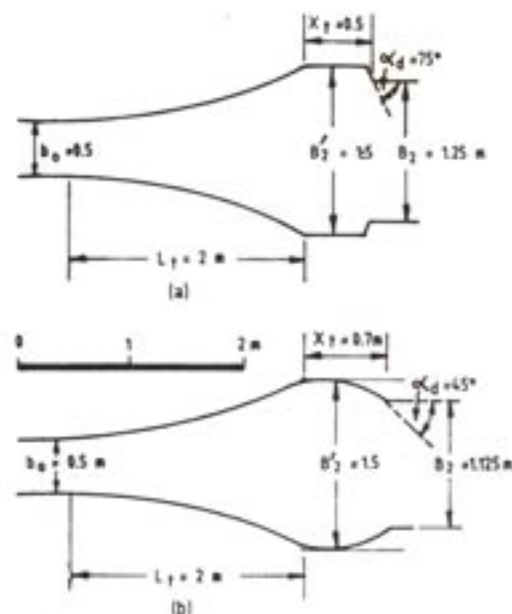


Figure 6: Rouse Modified Expansion ($F_D = 1$) Indicating (a) Abrupt Flow Deflector (b) Cusp-Type Flow Deflector

(d) **Rouse's Reverse Expansion**: Two different lengths of Rouse's Reverse expansion corresponding to design Froude's number $F_D = 1$ (Rouse R_1) and 4, (Rouse R_4) were tested (Figure 4). For $F_D = 4$, full length of Rouse's expansion is 15 m corresponding to expansion ratio of 3. As the flume length was 7.5 m only, results for $F_o = 4$ at $x = 7$ m correspond to an expansion width of 132 cm i.e. 88% diffusion of flow. However, the results obtained at $x = 7$ m provide an useful comparison regarding effectiveness of the various shock control devices vis-a-vis Rouse's design. While the longer length (Rouse R_4) was tested only for $F_o = 4$, the shorter one (Rouse R_1) was subject to flows with F_o

= 2, 4, 6 and 8. Results are furnished in Table 1 for $F_o = 4$ only. There was no shock wave for $F_o < F_D$ and the performance was extremely good. However, at Froude's number (F_o) higher than design F_D , shock waves re-appeared. Further details about Rouse's expansion are published in reference (Mazumder and Hager, 1993).

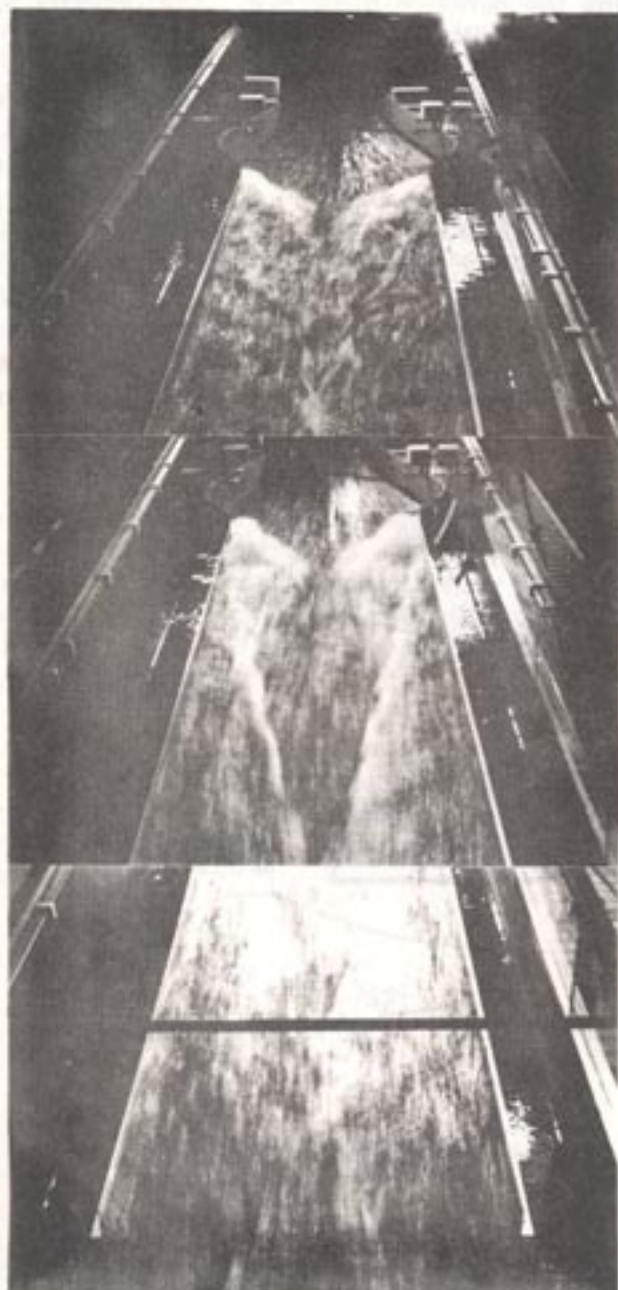


Figure 7: Flow pattern with Cusp Type Flow Deflector. $r=0.25$, $h_c=48$ mm $\alpha_c=45^\circ$, $x_c/L_c=0.35$. (a) $F_o=6$. (b) & (c) $F_o=8$.

(c) **Rouse's only Reverse curve with contraction at entry (Entry Contr. Rouse OR₁):** Since the expansion reach before the reverse curve was responsible for generation of shock waves downstream, it was decided to remove this reach by extending the reverse curve upto $x = 0$ ($b_c = 22$ cm). A straight contraction, as shown in Figure 8 was introduced

before the entry to expansion. The wall angle (θ) of the contraction was calculated so that the positive shock fronts generated in the contraction met the expansion walls tangentially at entry. The angle (19.5°) of expansion walls (at entry) was made equal to the angle of inclination of the shock fronts (β). For $\beta = 19.5^\circ$, θ - value was found to be 7.6° from Ippen's (1951) equation(s) with correction.

$$\tan \theta = \frac{\tan \beta \left(\sqrt{1 + 8 F_o^2 \sin^2 \beta} - 3 \right)}{2 \tan^2 \beta - 1 + \sqrt{1 + 8 F_o^2 \sin^2 \beta}} \quad \dots(5)$$

Correction, as obtained by Ippen from experiments, was applied to θ found from equation (5). Results obtained with the above mentioned modification of Rouse's design are given in Table 2. Flow patterns are shown in Figures 9(a), (b) and (c). Shock waves were completely eliminated at $Q=114$ Ls^{-1} ($F_o=2.5$) when the flow was choked at the entry Figure 9(a) and 9(b). With increasing values of Q (or F_o), shock fronts appeared but the length of the channel affected by shock wave was reduced considerably. Upto $Q = 140$ Ls^{-1} ($F_o = 3$), shock fronts, which started at the entry, ended within the expansion reach (Figure 9(c)). For $F_o > 4$, the meeting point moved downstream of expansion but the affected area bounded by the shock fronts and the expanding jet was reduced considerably as compared to Rouse's design for the corresponding F_o - value. There was a change in the orientation of Prandtl-Meyer shock fan due to contraction resulting in early restoration of normal flow.

SUMMARY OF RESULTS AND CONCLUSIONS

Performances of different types of supercritical expansion were measured with and without shock control devices. Results are furnished in terms of the parameters (W_p , α , T_p and $\Delta E/E_1$) in Tables 1 and 2, which give a visual comparison of the different performance criteria (for $F_o = 4$ only) at the end of the channel ($x=7m$). Shock waves were controlled/eliminated as apparent from a comparison of the Figures 2(a), (b), (c) (without shock control) with Figures 5, 7 and 9 with shock control measure). Improved performance of the expansion with different shock control devices are also apparent from a comparison of Tables 1 and 2. Based on the results obtained, the following conclusions may be drawn:

- Shock waves propagate downstream from the exit end of straight and curved expansion (Rouse M_1 and Rouse R_1) and the performance of the expansion is not satisfactory.
- Rouse's expansion with reverse curve is efficient in controlling shock waves as long as the Froude's number (F_o) of incoming flow does not exceed the design Froude's number F_D .

TABLE-1

Comparison of performances
 $x=7.00m$
 $Fo=4$
 Without shock control devices

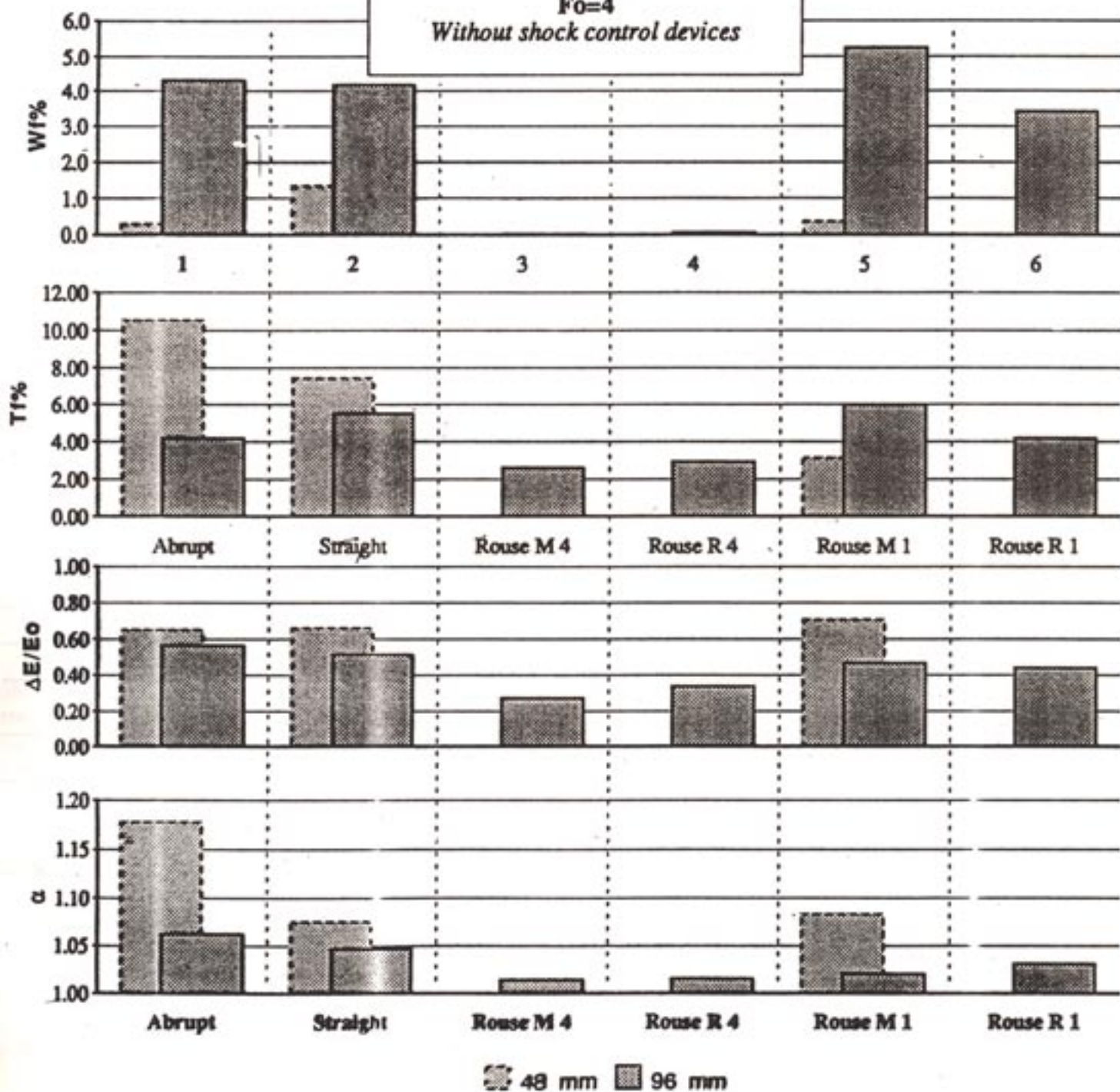
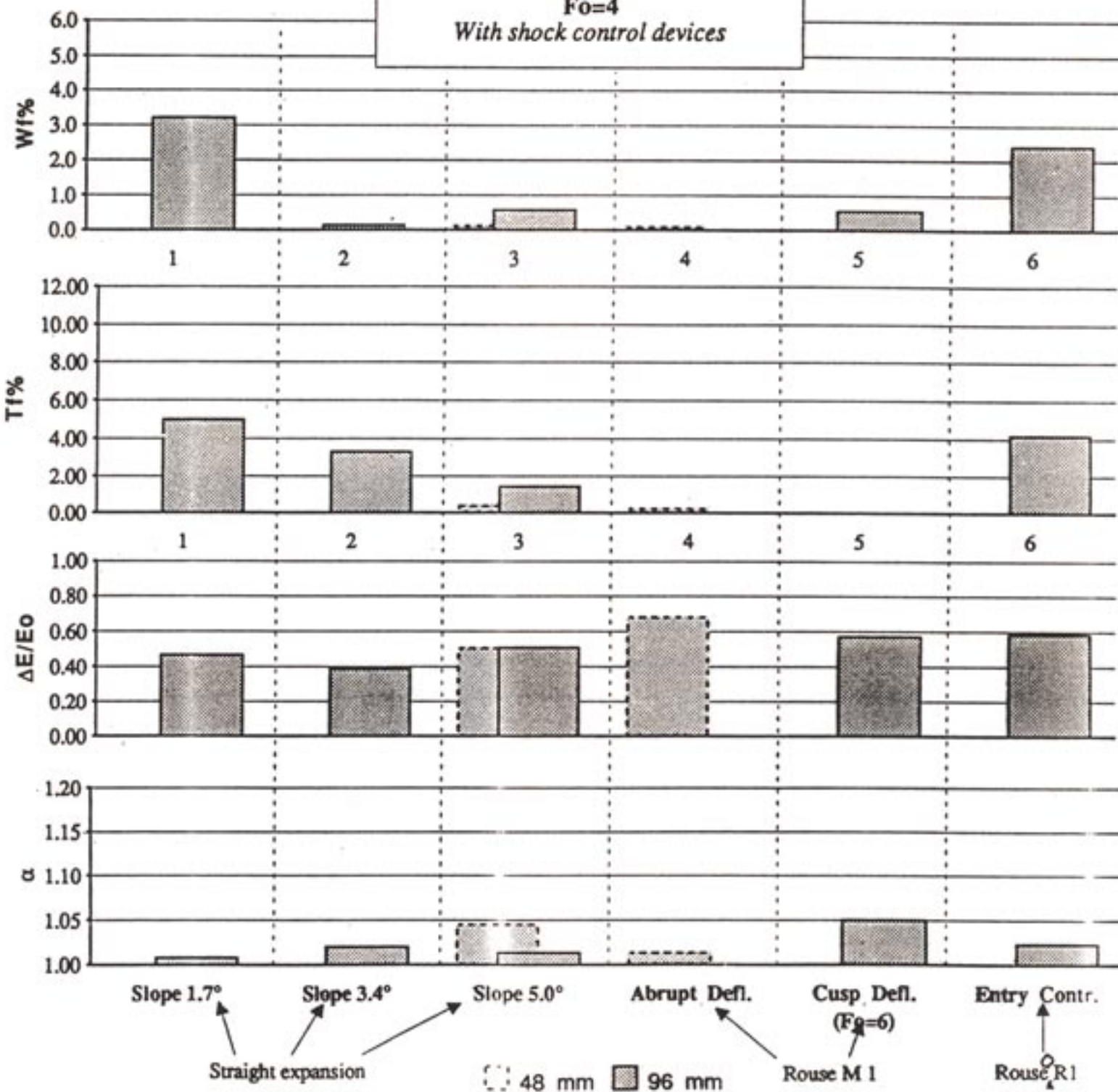


TABLE-2

Comparison of performances
 $x=7.00m$
 $Fo=4$
 With shock control devices



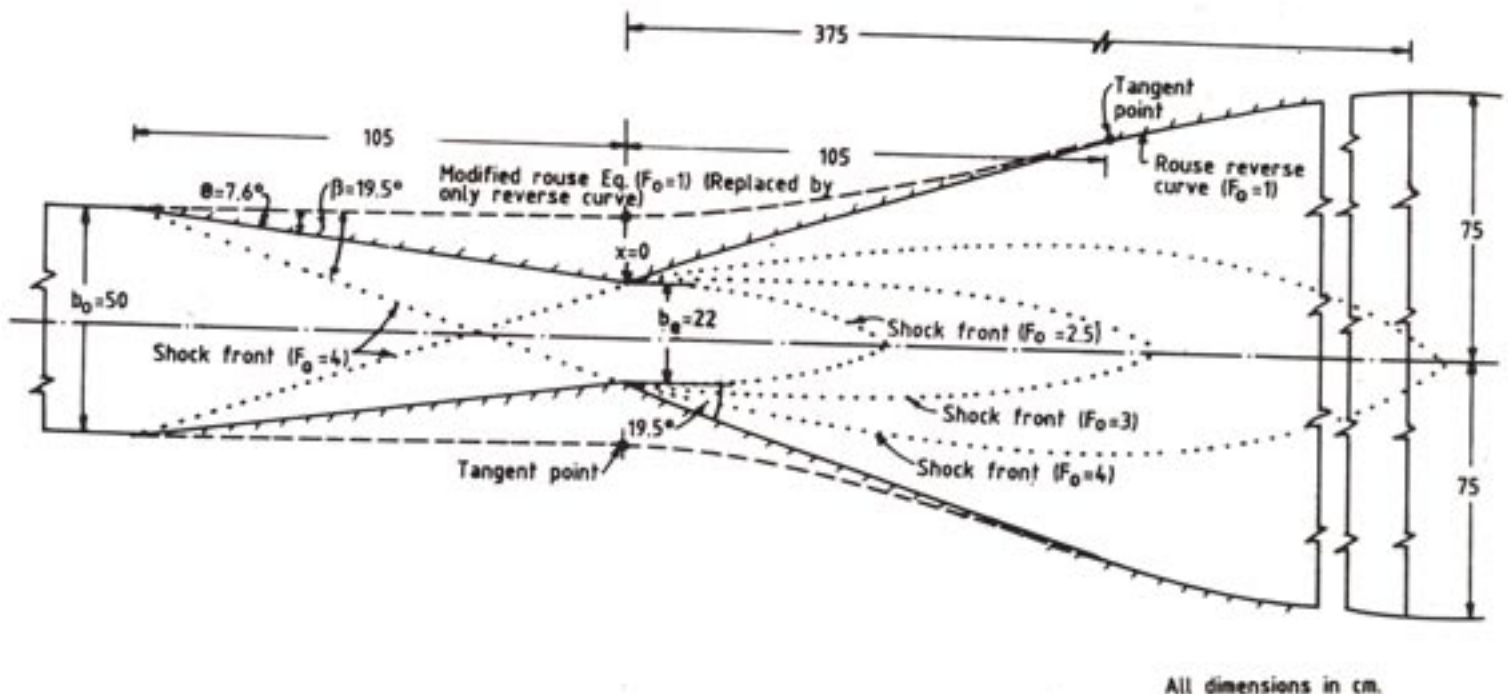


Figure 8: Rouse only Reverse Curve with Contraction Before Entry (Rouse OR₁, Entry Contr.).

3. It is possible to control shock waves by use of devices such as adverse slope, flow deflector and introduction of a properly designed contraction before Rouse's only reverse curve, even though the length of the expansion is considerably less compared to Rouse's classical design.

4. Choking of flow is an efficient means to control/eliminate shock waves. If the choking is done by providing a contraction near entry, it is essential that the exit of the expansion must be streamlined (as in Rouse's reverse curve); otherwise shock waves will reappear at the exit. If the choking is done by use of adverse slope with control at the exit, the shape of expansion boundary is immaterial. Even a straight expansion will be effective.

5. Straight expansion with proper adverse bed slope is efficient in stabilizing hydraulic jump inside the expansion. The flow becomes more axial, and the shock wave is reduced. Shock waves were completely

eliminated when the flow got choked.

6. More experiments are needed to establish the various geometries of the cusp-type flow deflector which was found to be effective in controlling shock waves.

7. Contraction, just before the commencement of expansion, changes the orientation of Prandtl-Meyer fan apart from generating positive shock waves which meet the expansion walls tangentially at entry. This is an effective method of controlling shock wave in supercritical expansion.

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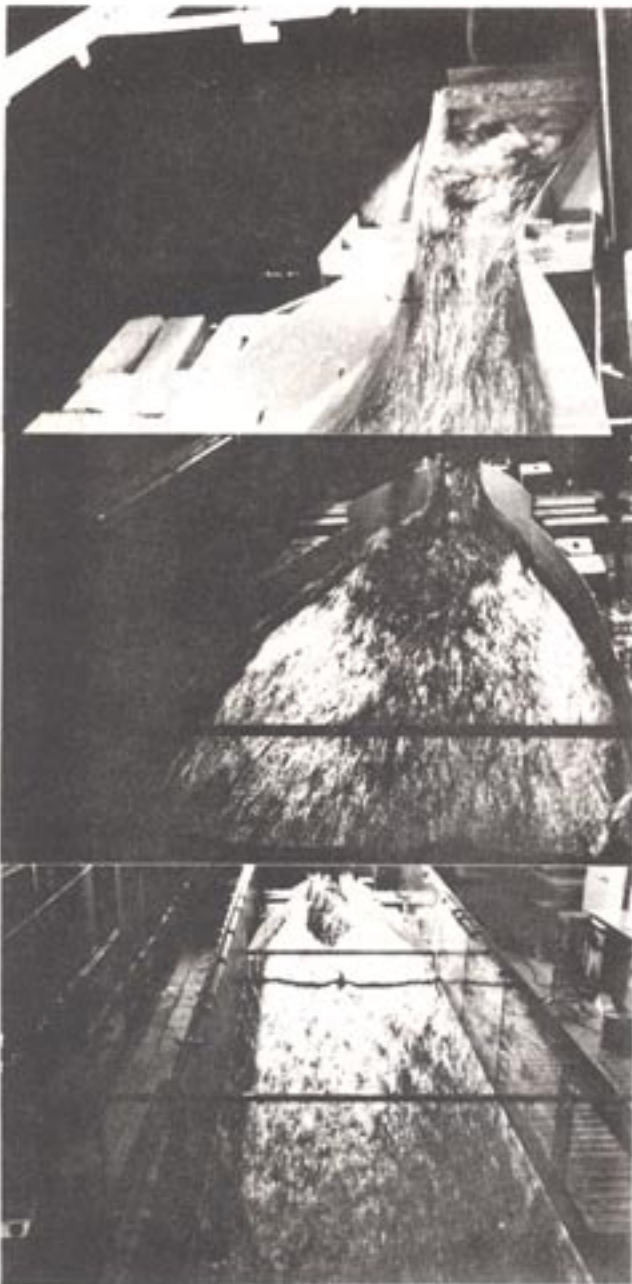


Figure 9: Flow pattern in Rouse OR₁, Entry Contr., $h_s=48$ mm
 (a) Choked Flow at $F_s=2.45$ (b) Complete elimination of shockwaves downstream of Expansion with choking, $F_s=2.45$ (c) Shock front reappears but area affected by shockwave is very small.